

CS 173, Spring 2015**Examlet 2, Part A****NETID:****FIRST:****LAST:****Discussion: Monday 9 10 11 12 1 2 3 4 5**

Prove the following claim, using your best mathematical style and the following definition of congruence mod k : $a \equiv b \pmod{k}$ if and only if $a - b = nk$ for some integer n .

Claim: For all integers a, b, c, d, j and k (j and k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ and $j|k$, then $a + c \equiv b + d \pmod{j}$.

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Recall that $\gcd m, n$ is the largest integer that divides both m and n . Use this definition and your best mathematical style to prove the following claim by contrapositive.

For all integers p and q , if $p + 6q = 23$ then $\gcd(p, q) \neq 7$.

Begin by explicitly stating the contrapositive of the claim:

Now prove the contrapositive:

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Prove the following claim, using your best mathematical style and the following definition of congruence mod k : $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n .

For all integers a, b, p, q and k (k positive), if $a \equiv b \pmod{2k}$ and $p \equiv q \pmod{k}$, then $a(p+1) \equiv b(q+1) \pmod{k}$.

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For all integers a, b, c, p and k (c positive), if $ap \equiv b \pmod{c}$ and $k \mid a$ and $k \mid c$, then $k \mid b$.

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Prove the following claim, using your best mathematical style and the following definition of congruence mod k : $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n .

For all integers x, y, p, q and m , with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $x^2 + xy \equiv p^2 + pq \pmod{m}$.

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Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim by contrapositive.

For all real numbers x and y , if x is not rational, then $2x + 3y$ is not rational or y is not rational.

Begin by explicitly stating the contrapositive of the claim:

Now prove the contrapositive: