\mathbf{CS}	173,	\mathbf{Sp}	ring	2015
Exa	\mathbf{mlet}	2,	Part	$\mathbf{t} \; \mathbf{B}$

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Discussion:

Monday

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1. (5 points) Let a and b be integers, b > 0. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is a = bq + r. What is the other?

Solution: $0 \le r < b$

2. (6 points) Use the Euclidean algorithm to compute gcd(1702, 1221). Show your work.

Solution:

1702 - 1221 = 481

$$1221 - 481 \times 2 = 1221 - 962 = 259$$

481 - 259 = 222

259 - 222 = 37

 $222 - 6 \times 37 = 0$

So gcd(1702, 1221) = 37

 $\gcd(1702, 1221).$

3. (4 points) Check the (single) box that best characterizes each item.

 $-7 \equiv 13 \pmod{5}$

true

false

 $gcd(p,q) = \frac{pq}{lcm(p,q)}$ (p and q positive integers)

true for all p, q

true for some p, q

true for p, q prime

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Claim: For all positive integers a, b, and c, if gcd(a,bc) > 1, then gcd(a,b) > 1 and gcd(a,c) > 1.

Solution: This is false. Consider a = b = 3 and c = 2. Then bc = 6. So gcd(a, bc) = 3 > 1 but gcd(a, c) = 1.

2. (6 points) Use the Euclidean algorithm to compute gcd(1012, 299). Show your work.

Solution:

$$1012 - 3 \times 299 = 1012 - 897 = 115$$

 $299 - 2 \times 115 = 299 - 230 = 69$
 $115 - 69 = 46$
 $69 - 46 = 23$
 $46 - 2 \times 23 = 0$
So $gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

Zero is a multiple of 7.	true $\sqrt{}$	false
For any positive integers p and q ,		
if $lcm(p,q) = pq$, then p and q are relatively prime.	true $\sqrt{}$	false

CS 173, Sp Examlet 2,	•	5 N	ETID:									
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1. (5 points) Let a remainder r of a Solution: $0 \le a$	a divided by b .	_									quotient q an	nd the
2. (6 points) Use to Solution: $2262 - 546 \times 4 = 546 - 7 \times 78 = 50 \text{ gcd}(2262, 546)$	he Euclidean a = 2262 – 2184 0		hm to co	omp	oute gco	1(2262	2, 546). Sh	ow y	our	work.	
3. (4 points) Check	k the (single)	box th	at best c	char	acteriz	es eac	ch ite	m.				
$k \equiv -k \pmod{7}$)		e for all <i>k</i> e for all <i>k</i>	ľ		tr	rue fo	r son	ne k	V	/	
For all prime nutwo natural num				${ m tr}$	rue ,	/	fa	alse				

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1. (5 points) Is the forexample showing the	~	ie? Informa	ally expla	in why i	it is, or a	give a cond	crete counter-
Claim: For all	non-zero integers	a and b , if	$a \mid b$ and	$b \mid a$, th	en $a=b$.		
Solution: This is f	Talse. Consider $a =$	= 3 and $b =$	= -3. The	en $a \mid b$ a	$nd b \mid a$,	but $a \neq b$.	
2. (6 points) Use the l	Euclidean algorith	nm to comp	ute gcd(1	568, 546). Show y	your work.	
Solution:							
$1568 - 546 \times 2 = 1$	568 - 1092 = 476						
546 - 476 = 70							
$476 - 70 \times 6 = 476$	-420 = 56						
70 - 56 = 14							
$56 - 14 \times 3 = 0$							
So the GCD is 14.							
3. (4 points) Check th	e (single) box tha	at best char	acterizes	each iter	n.		
$\gcd(0,0)$	0	1	infinite		undef	ined $\sqrt{}$	
If a and b are posit	ive integers and						

true

false

r = remainder(a, b),then gcd(b, r) = gcd(a, b)

	CS 173, Spring 2015 Examlet 2, Part B
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Ι	Discussion: Monday 9 10 11 12 1 2 3 4 5
1.	(5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.
	Claim: For any positive integers p and q , $p \equiv q \pmod{1}$. Solution: This is true. $p \equiv q \pmod{1}$ is equivalent to $p - q = n \times 1 = n$ for some integer n . But
	we can always find an integer that will make this equation balance!
2.	(6 points) Use the Euclidean algorithm to compute $\gcd(1495,221)$. Show your work.
	Solution:
	$1495 = 221 \times 6 = 1495 - 1326 = 169$
	$221 - 169 = 52$ $169 - 52 \times 3 = 169 - 156 = 13$
	$52 \times 3 = 103$ $100 = 13$ $52 - 13 \times 4 = 0$
	So $\gcd(1495, 221) = 13$.
3.	(4 points) Check the (single) box that best characterizes each item.
	If p, q , and k are positive integers, then $\gcd(pq, qk) = q \square pq \square pqk \square q\gcd(p, k) $

true

false

Two positive integers p and q are relatively prime if and only if gcd(p,q)=1.

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1. (5 points) Is the example showing	he following cla ng that it is not		rue? I	nform	ally exp	olain	why	it is,	or g	ive a (concret	e cou	inte

For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $p \mid q$, then $s \equiv t \pmod{q}$.

Solution: Consider s = 1, t = 4, p = 3 and q = 6. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute gcd(221, 1224). Show your work.

Solution:

$$1224 - 5 \times 221 = 1224 - 1105 = 119$$

 $221 - 119 = 102$
 $119 - 102 = 17$
 $102 - 17 \times 6 = 0$
So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

