CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST: **LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution:

   $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So $\gcd(1702, 1221) = 37$

   $\gcd(1702, 1221)$.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod 5$ — true ☑ false ☐

   $\gcd(p, q) = \dfrac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) — true for all p, q ☑ true for some p, q ☐ true for p, q prime ☐

CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST:	**LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ and $\gcd(a, c) > 1$.

   Solution: This is false. Consider $a = b = 3$ and $c = 2$. Then $bc = 6$. So $\gcd(a, bc) = 3 > 1$ but $\gcd(a, c) = 1$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1012, 299)$. Show your work.

   Solution:

   $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So $\gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

Zero is a multiple of 7.	true ☑ $\surd$	false ☐
For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime.	true ☑ $\surd$	false ☐

CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2262, 546)$. Show your work.

   Solution:

   $2262 - 546 \times 4 = 2262 - 2184 = 78$

   $546 - 7 \times 78 = 0$

   So $\gcd(2262, 546) = 78$

3. (4 points) Check the (single) box that best characterizes each item.

   $k \equiv -k \pmod 7$ — true for all k ☐ — true for some k ☑ — false for all k ☐

   For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$. — true ☑ — false ☐

CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is false. Consider $a = 3$ and $b = -3$. Then $a \mid b$ and $b \mid a$, but $a \neq b$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   gcd(0,0) — 0 ☐ 1 ☐ infinite ☐ undefined ☑ (√)

   If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(a, b)$ — true ☑ (√) false ☐

CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any positive integers p and q, $p \equiv q \pmod 1$.

   Solution: This is true. $p \equiv q \pmod 1$ is equivalent to $p - q = n \times 1 = n$ for some integer n. But we can always find an integer that will make this equation balance!

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1495, 221)$. Show your work.

   Solution:

   $1495 = 221 \times 6 = 1495 - 1326 = 169$

   $221 - 169 = 52$

   $169 - 52 \times 3 = 169 - 156 = 13$

   $52 - 13 \times 4 = 0$

   So $\gcd(1495, 221) = 13$.

3. (4 points) Check the (single) box that best characterizes each item.

   If p, q, and k are positive integers, then $\gcd(pq, qk) =$ q ☐ pq ☐ pqk ☐ $q\gcd(p,k)$ ☑

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$. true ☑ false ☐

CS 173, Spring 2015
Examlet 2, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $p \mid q$, then $s \equiv t \pmod{q}$.

   Solution: Consider $s = 1$, $t = 4$, $p = 3$ and $q = 6$. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(221, 1224)$. Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $25 \equiv 4 \pmod{7}$ true [√] false [ ]

   gcd(k,0) 0 [ ] k [√] undefined [ ]