CS 173, Spring 2015
Examlet 3, Part B

NETID:

FIRST:

LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$ $C = \{3, 7\}$

   $A \times (B \cap C) = A \times \{3\} = \{(\text{fox}, 3), (\text{cat}, 3)\}$

   $A \cap B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \in A$

   true for all sets A ☐ true for some sets A ☑

   false for all sets A ☐

   If $x \in A \cap B$, then $x \in A$.

   true for all sets A and B ☑ true for some sets A and B ☐

   false for all sets A and B ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^6 + [5]^3$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^6 = [3]^3 = [27] = [5]$

   $[5]^3 = [125] = [4]$

   $[6]^6 + [5]^3 = [5] + [4] = [9]$

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1. (4 points) $A = \{\text{fox, tiger, wolf}\}$ $B = \{3, 4\}$ $C = \{6, 7, 8\}$

   $| A \times (B \cup C) | = |\text{fox, tiger, wolf} \times \{3, 4, 6, 7, 9\}| = 3 \times 5 = 15$

   $\{p + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 3 \text{ and } 1 \leq q \leq 3\} = \{2, 3, 4, 5, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap (B \cup C)$ $= (A \cap B) \cup (A \cap C)$

   true for all sets A [√] true for some sets A [ ]

   false for all sets A [ ]

   $\emptyset$ is

   an element of $\mathbb{Z}$ [ ] a subset of $\mathbb{Z}$ [√]

   both [ ] neither [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{22} = [8]^{16} \cdot [8]^4 \cdot [8]^2 = [3][4][9]$

   $[3][4][9] = [3][36] = [3][3] = [9]$

   So $[8]^{22} = [9]$

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1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{\text{cat}, \text{mouse}\}$

   $A \cap B = \{\text{cat}\}$

   $\{p + q^2 \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} = \{2, 3, 5, 6, 10, 11\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \times \emptyset =$ $\emptyset$ [√] $\{\emptyset\}$ [ ] $\{\emptyset, \emptyset\}$ [ ] $\{(\emptyset, \emptyset)\}$ [ ]

   $|A - B| = |A| - |B|$ true for all sets A [ ] true for some sets A [√]
   false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution:

   $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{21} = [5]^{16} \cdot [5]^4 \cdot [5] = [4][4][5] = [80] = [8]$

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1. (4 points) $A = \{4, 5, 9\}$ $B = \{\text{arya}, \text{bran}\}$ $C = \{\ 2, 4, 10\ \}$

   $B \times A = \{\ (\text{arya}, 4), (\text{arya}, 5), (\text{arya}, 9), (\text{bran}, 4), (\text{bran}, 5), (\text{bran}, 9)\}$

   $A \cap C = \{4\}$

2. (4 points) Check the (single) box that best characterizes each item.

   Sets A and B are disjoint

$A - B = B - A$	☐	$A = \overline{B}$	☐
$A \cap B = \{\emptyset\}$	☐	$A \cap B = \emptyset$	☑

   $\{13, 14, 15\} \times \emptyset =$

$\emptyset$	☑	$\{\emptyset\}$	☐	$\{13, 14, 15\}$	☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9][10][7]$

   $[9][10][7] = [90][7] = [-1][7] = [-7] = [6]$

   So $[7]^{19} = [6]$

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1. (4 points) $A = \{\text{apple}, \text{lemon}\}$ $B = \{4, 5, 9\}$ $C = \{\ (\text{apple}, 4),\ (5, \text{lemon})\ \}$

   $\emptyset \times B = \emptyset$

   $(A \times B) \cap C = \{(\text{apple}, 4)\}$

2. (4 points) Check the (single) box that best characterizes each item.

$\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert$	true for all sets A	☐	true for some sets A	☑
	false for all sets A	☐		
$\emptyset \in A$	true for all sets A	☐	true for some sets A	☑
	false for all sets A	☐		

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{15}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution: $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^{15} = [7]^8[7]^4[7]^2[7] = [-2][3][5][7] = [-6][35] = [5][2] = [10]$

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1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\text{ cup}, \text{mug }\}$ $C = \{\text{wine}, (\text{water}, \text{beer})\}$

   $A \times B = \{\ (\text{water}, \text{cup}), (\text{beer}, \text{cup}), (\text{wine}, \text{cup}), (\text{water}, \text{mug}), (\text{beer}, \text{mug}), (\text{wine}, \text{mug})\ \}$

   $A \cap C = \{\text{wine}\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\{1,2\} \cup \emptyset =$

$\emptyset$	☐	$\{\emptyset\}$	☐	$\{1,2\}$	☑
$\{(1,\emptyset),(2,\emptyset)\}$	☐	$\{1,2,\emptyset\}$	☐	undefined	☐

   $A \times B = B \times A$

true for all sets A and B	☐	false for all sets A and B	☐
true for some sets A and B	☑		

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{18}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^6 = ([7]^2)^3 = [-3]^3 = [-27] = [-1]$

   $[7]^{18} = ([7]^6)^3 = [-1]^3 = [-1] = [12]$