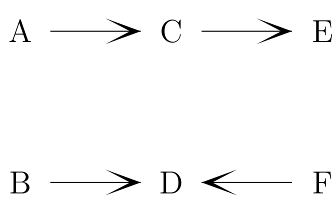
CS 173, Spring 2015
Examlet 4, Part B

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: ☐ Irreflexive: ☑ $\surd$

Symmetric: ☐ Antisymmetric: ☑ $\surd$

Transitive: ☐

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, antisymmetric, transitive

3. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give five members of the equivalence class $[13]$.

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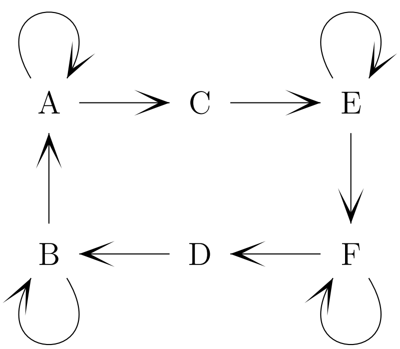

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ √

Transitive: ☐

2. (5 points) Suppose that R is a partial order on a set A. What additional property is required for R to be a linear order (aka total order)? Give specific details of the property, not just its name.

   Solution: all pairs of elements must be comparable. That is, for any elements x and y in A, either xRy or yRx.

3. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(x, y) \sim (p, q)$ if and only $|x| + |y| = |p| + |q|$. List three members of $[(2, 3)]$.

   Solution: (2,3), (-2,3), (1, -4)

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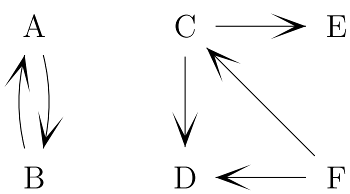
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

   Solution: For any $x, y \in A$, if xRy and yRx, then $x = y$. Or for any $x, y \in A$, if xRy and $x \neq y$, then $y \not R x$.

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. Consider $(1, 2)$, $(2, 3)$, and $(3, 4)$. Then $(1, 2)T(2, 3)$ and $(2, 3)T(3, 4)$, but not $(1, 2)T(3, 4)$.

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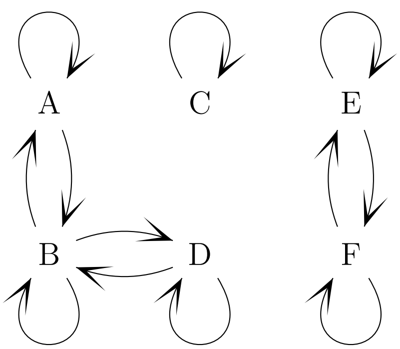

Reflexive: [√] Irreflexive: [ ]

Symmetric: [√] Antisymmetric: [ ]

Transitive: [ ]

2. (5 points) A relation is an equivalence relation if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, symmetric, transitive

3. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(a, b) \sim (p, q)$ if and only $ab = pq$. List three members of $[(5, 6)]$.

   Solution: (5,6), (1,30), (-15,-2)

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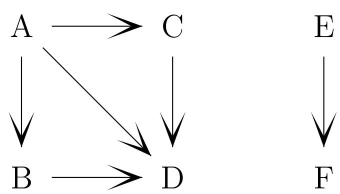
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: [ ] Irreflexive: [√]

Symmetric: [ ] Antisymmetric: [√]

Transitive: [√]

2. (5 points) A relation is a strict partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: irreflexive, antisymmetric, transitive

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "disjoint" relation D on J by $(a, b)D(c, d)$ if and only if $b \leq c$ or $d \leq a$. Is D transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Typo: The original version had $(a, b)T(c, d)$ which was corrected to $(a, b)D(c, d)$ at the exam.

   Solution: D is not transitive. Consider $(1, 2)$, $(3, 5)$, and $(4, 6)$. Then $(1, 2)D(3, 5)$ and $(3, 5)D(4, 6)$. But not $(1, 2)D(4, 6)$.

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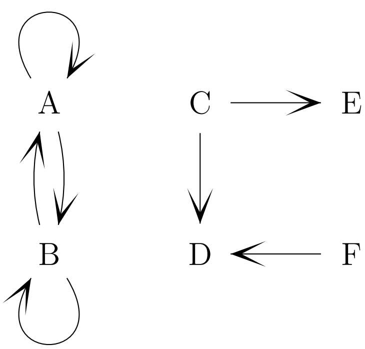

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☑

2. (5 points) Suppose that R is an equivalence relation on a set A. Using precise set notation, define the equivalence class $[x]_R$.

   Solution: $[x]_R = \{y \in A \mid xRy\}$

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $\gcd(a, b) > 1$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. Consider 2, 6, and 3. Then $\gcd(2, 6) > 1$ and $\gcd(6, 3) > 1$, but $\gcd(2, 3) = 1$.