CS 173, Spring 2015
Examlet 5, Part A

NETID:	

FIRST:	LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (f(x) - y, 5y + 3)$. Prove that g is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

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1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define the function $f : (0, \infty) \to \left(\frac{1}{3}, \infty\right)$ by $f(x) = \dfrac{x^2 + 2}{3x^2}$. Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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1. (10 points) Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (h(x) - y, 3h(x) + 1)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Suppose that $g : A \to B$ and $f : B \to C$. Prof. Snape claims that if $f \circ g$ is one-to-one, then f is one-to-one. Disprove this claim using a concrete counter-example in which A, B, and C are all small finite sets.

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1. (10 points) Suppose that $g : \mathbb{N} \to \mathbb{N}$ is one-to-one. Let's define the function $f : \mathbb{N}^2 \to \mathbb{N}^2$ by the equation $f(x, y) = (x + g(y), g(x))$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

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1. (10 points) Let $g : \mathbb{N} \to \mathbb{N}$ be onto, and let $f : \mathbb{N}^2 \to \mathbb{Z}$ be defined by

$$f(n, m) = (m - 1)g(n)$$

Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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1. (10 points) Suppose that $g : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $h : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $h(x, y) = (g(x) + g(y), g(x) - g(y))$. Prove that h is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.