CS 173, Spring 2015
Examlet 5, Part B

NETID:

FIRST:

LAST:

Discussion: Monday   9   10   11   12   1   2   3   4   5

1. (5 points) How many different 13-letter strings can be made be rearranging the characters in the word ''massachusetts''? Show your work.

   Solution: There are 13 letters total, with 4 copies of s, two t's, and 2 a's. So the number of possibilities is

   $$\frac{13!}{4!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices.   true ☐   false ☑

   $f : \mathbb{N} \to \mathbb{R},\ f(x) = x^2 + 2$

   onto ☐   not onto ☑

   not a valid function ☐

   $f : \mathbb{N}^2 \to \mathbb{N},\ f(p, q) = pq$

   onto ☑   not onto ☐

   not a valid function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2,\ g(x, y) = (y, 3x)$

   one-to-one ☑   not one-to-one ☐

   not a valid function ☐

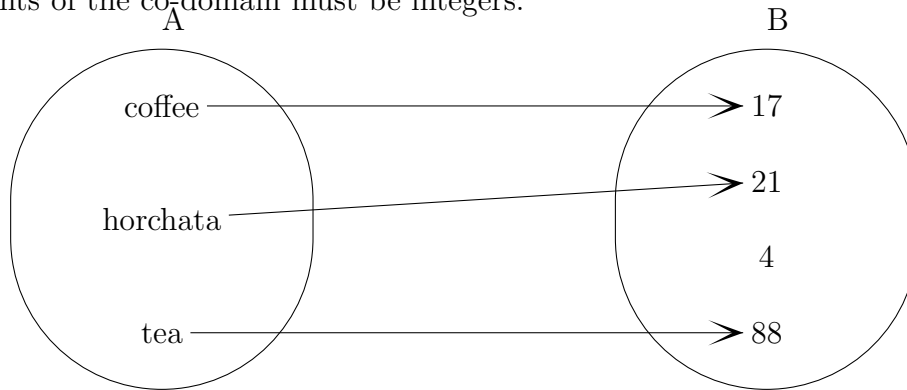 $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$ true ☐   false ☑

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1. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the co-domain and arrows showing how input values map to output values. The elements of the co-domain must be integers.

A

B

coffee —————————————————→ 17

horchata —————————→ 21

4

tea —————————————→ 88

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one.

true ☐   false ☑

$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ if x is even, and $f(x) = x - 22$ if x is odd

onto ☐   not onto ☑

not a valid function ☐

$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \left\lfloor \frac{x}{3} \right\rfloor$

onto ☑   not onto ☐

not a valid function ☐

$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = |x|$

one-to-one ☐   not one-to-one ☑

not a valid function ☐

$\exists y \in \mathbb{N}, \forall x \in \mathbb{N}, x = xy$

true ☑   false ☐

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1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

   Solution: q^p

2. (10 points) Check the (single) box that best characterizes each item.

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices.

   true ☐   false ☑

   $f : \mathbb{N} \to \mathbb{R}, \ f(x) = x^2 + 2$

   onto ☐   not onto ☑

   not a valid function ☐

   $f : \mathbb{N}^2 \to \mathbb{N}, \ f(p, q) = pq$

   onto ☑   not onto ☐

   not a valid function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2, \ g(x, y) = (y, 3x)$

   one-to-one ☑   not one-to-one ☐

   not a valid function ☐

   $\exists y \in \mathbb{Z}, \ \forall x \in \mathbb{Z}, \ y \leq x$

   true ☐   false ☑

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ''minimalist''? Show your work.

   Solution: There are 10 characters total, with two copies of m and three copies of i. So the total number of permutations is

   $$\frac{10!}{3!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is

   the domain of f ☐   the co-domain of f ☐

   the image of f ☑

   $g : \mathbb{Z} \to \mathbb{N}$, $g(x) = |x|$

   onto ☑   not onto ☐

   not a valid function ☐

   $f : \mathbb{N} \to \mathbb{N}$, $f(x) = 3 - x$

   one-to-one ☐   not one-to-one ☐

   not a valid function ☑

   $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ if x is even, and $f(x) = x - 22$ if x is odd

   one-to-one ☐   not one-to-one ☑

   not a valid function ☐

   $\exists y \in \mathbb{R}^+, \forall x \in \mathbb{R}^+, xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)

   true ☐   false ☑

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1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

   Solution: $\frac{q!}{(q-p)!}$

2. (10 points) Check the (single) box that best characterizes each item.

   A function is onto if and only if its image is the same as its co-domain.   true $\boxed{\checkmark}$ false $\square$

   $f : \mathbb{Z} \to \mathbb{R}, \ f(x) = x$

   one-to-one $\boxed{\checkmark}$ not one-to-one $\square$

   not a valid function $\square$

   $g : \mathbb{Z} \to \mathbb{Z}, \ g(x) = 7 - \left\lfloor \frac{x}{3} \right\rfloor$

   one-to-one $\square$ not one-to-one $\boxed{\checkmark}$

   not a valid function $\square$

   $f : \mathbb{N}^2 \to \mathbb{Z}, \ f(p, q) = 2^p 3^q$

   onto $\square$ not onto $\boxed{\checkmark}$

   not a valid function $\square$

   $\forall x \in \mathbb{Z}, \ \exists y \in \mathbb{Z}, \ x \neq y$ and $x + y = 0$ true $\square$ false $\boxed{\checkmark}$

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

   Solution: There are qn elements in $B \times C$. So there are $(qn)^p$ ways to build a function from A to $B \times C$.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : A \to B$ is onto, then $|A| \geq |B|$ ☑ $|A| \leq |B|$ ☐ $|A| = |B|$ ☐

   $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = x$

   one-to-one ☐   not one-to-one ☐

   not a valid function ☑

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$

   one-to-one ☑   not one-to-one ☐

   not a valid function ☐

   $g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$

   onto ☐   not onto ☐

   not a valid function ☑

   $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+, xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)   true ☑   false ☐