CS 173, Spring 2015
Examlet 5, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) How many different 13-letter strings can be made be rearranging the characters in the word ``massachusetts''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☐

$f : \mathbb{N} \to \mathbb{R},\ f(x) = x^2 + 2$ onto ☐ not onto ☐ not a valid function ☐

$f : \mathbb{N}^2 \to \mathbb{N},\ f(p, q) = pq$ onto ☐ not onto ☐ not a valid function ☐

$g : \mathbb{R}^2 \to \mathbb{R}^2,\ g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$ true ☐ false ☐

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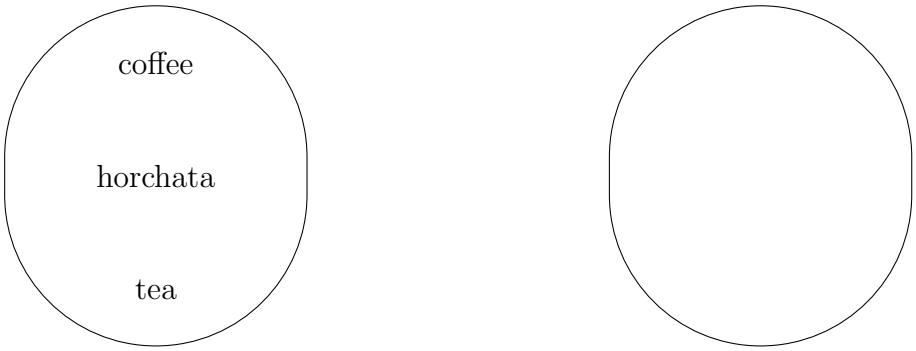
1. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the co-domain and arrows showing how input values map to output values. The elements of the co-domain must be integers.



2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false ☐

$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ if x is even, and $f(x) = x - 22$ if x is odd
onto ☐ not onto ☐ not a valid function ☐

$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$
onto ☐ not onto ☐ not a valid function ☐

$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = |x|$
one-to-one ☐ not one-to-one ☐ not a valid function ☐

$\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$ true ☐ false ☐

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1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☐

$f : \mathbb{N} \to \mathbb{R},\ f(x) = x^2 + 2$ onto ☐ not onto ☐ not a valid function ☐

$f : \mathbb{N}^2 \to \mathbb{N},\ f(p, q) = pq$ onto ☐ not onto ☐ not a valid function ☐

$g : \mathbb{R}^2 \to \mathbb{R}^2,\ g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$ true ☐ false ☐

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1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ‘‘`minimalist`’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is
the domain of f ☐ the co-domain of f ☐ the image of f ☐

$g : \mathbb{Z} \to \mathbb{N}$, $g(x) = |x|$
onto ☐ not onto ☐ not a valid function ☐

$f : \mathbb{N} \to \mathbb{N}$, $f(x) = 3 - x$
one-to-one ☐ not one-to-one ☐ not a valid function ☐

$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ if x is even, and $f(x) = x - 22$ if x is odd
one-to-one ☐ not one-to-one ☐ not a valid function ☐

$\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
($\mathbb{R}^+$ is the positive real numbers.)
true ☐ false ☐

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1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain. true ☐ false ☐

$f : \mathbb{Z} \to \mathbb{R},\ f(x) = x$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$g : \mathbb{Z} \to \mathbb{Z},\ g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$f : \mathbb{N}^2 \to \mathbb{Z},\ f(p, q) = 2^p 3^q$ onto ☐ not onto ☐ not a valid function ☐

$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x \neq y$ and $x + y = 0$ true ☐ false ☐

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is onto, then $|A| \geq |B|$ ☐ $|A| \leq |B|$ ☐ $|A| = |B|$ ☐

$f : \mathbb{R} \to \mathbb{Z},\ f(x) = x$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$g : \mathbb{N} \to \mathbb{Z},\ g(x) = x^2$ one-to-one ☐ not one-to-one ☐ not a valid function ☐

$g : \mathbb{Z} \to \mathbb{N},\ g(x) = x$ onto ☐ not onto ☐ not a valid function ☐

$\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}^+,\ xy = 1$
($\mathbb{R}^+$ is the positive real numbers.) true ☐ false ☐