CS 173, Spring 2015
Examlet 6, Part A

NETID:

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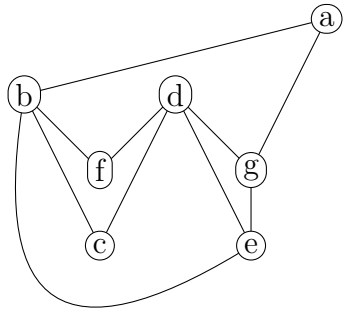
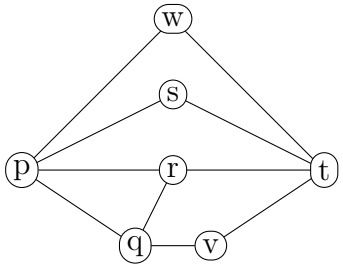
Discussion: Monday 9 10 11 12 1 2 3 4 5

Graph X



Graph Y



1. (11 points) Are graphs X and Y (above) isomorphic? Justify your answer.

   Solution: Yes. We can map the nodes as follows:

   f(b) = t, f(d) = p, f(f) = w, f(c) = s,, f(e) = r, f(g) = q), f(a) = v.

2. (4 points) Is C_5 a subgraph of W_7? Briefly justify your answer.

   Solution: Yes. The copy of C_5 consists of 4 consecutive nodes along the rim, plus the hub of the wheel. (A picture would also work here.)

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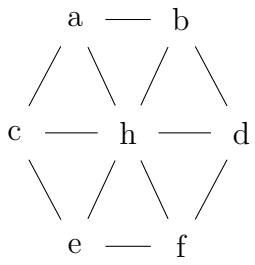
1. (11 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: 12. There are 6 choices for how to map node a. Then node b can map to either of the two adjacent nodes. After that, the rest of the mapping is forced.

2. (4 points) How many edges are in the complete bipartite graph $K_{11,6}$?

Solution: $11 \times 6 = 66$

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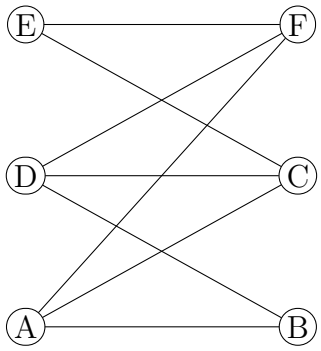
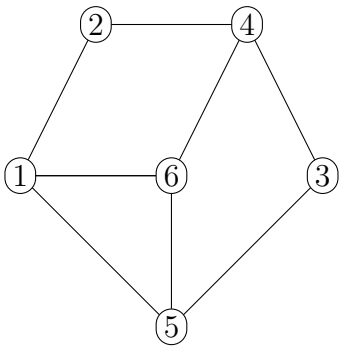
Graph X



Graph Y



1. (11 points) Are graphs X and Y (above) isomorphic? Justify your answer.

   Solution: No, they aren't isomorphic. Graph Y has a 5-cycle but graph X has only even cycles. Also, one of the degree-3 nodes in graph Y is connected to three degree-3 nodes, but in graph X each degree-3 node is connected to only two other degree-3 nodes.

2. (4 points) The complete graph K_8 contains 8 vertices. How many edges does it have?

   Solution: $\frac{8 \cdot 7}{2} = 28$

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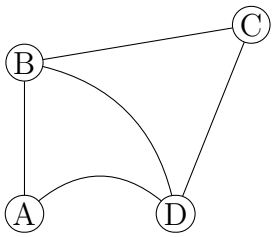

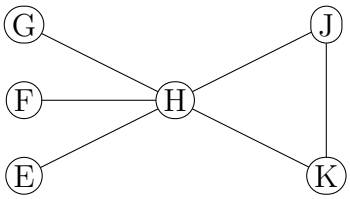

1. (11 points) How many isomorphisms are there from G (including all 10 nodes above) to itself? Justify your answer and/or show your work clearly .

   Solution: Nodes A and C can be interchanged, as can (independently) B and D. So there are four choices for the lefthand component. On the right size, J and K can be interchanged (2 choices) and E,F,G permuted (3! choices). So there are 12 choices for the righthand component. Thus a total of 48 isomorphisms of the whole graph.

2. (4 points) Is G bipartite? Briefly explain why or why not.

   Solution: No, it is not bipartite. It contains copies of C_3 (e.g. A,B,D) which cannot be divided between the two sets as required by the definition of bipartite.

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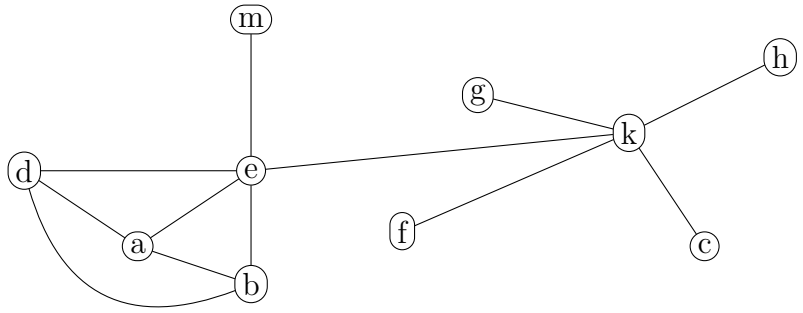
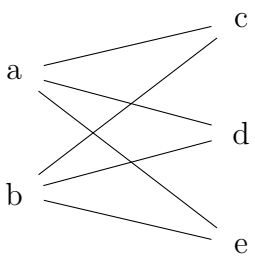


1. (11 points) How many isomorphisms are there from G (above) to itself? Justify your answer and/or show your work clearly .

   Solution: The four nodes f, g, c, h can be permuted (4! choices). independently, the nodes a, b, d can be permuted (3! choices). So there are a total of $4! \cdot 3! = 144$ isomorphisms.

2. (4 points) Draw a picture of the graph $K_{2,3}$.

   Solution:



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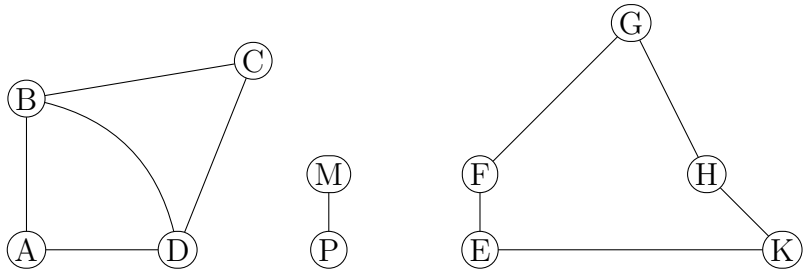

1. (11 points) How many isomorphisms are there from G (including all 11 nodes above) to itself? Justify your answer and/or show your work clearly .

   Solution: A and C can be swapped, as can (independently) B and D. So there are four choices for the lefthand component. M and P can be swapped (2 choices). For the righthand, there are 5 choices for where G maps to, then two choices for the image of H. So a total of 10 choices.

   So the total number of isomorphisms is $4 \cdot 2 \cdot 10 = 80$.

2. (4 points) Is the graph C_{10} bipartite? Briefly justify your answer.

   Solution: Yes, it is bipartite. As you walk around the cycle, assign nodes to the two subsets in an alternating manner.