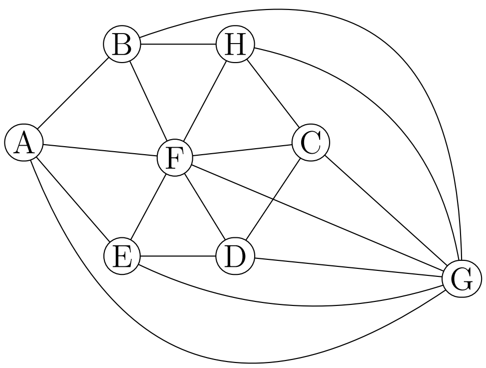
CS 173, Fall 2014
Examlet 7, Part B

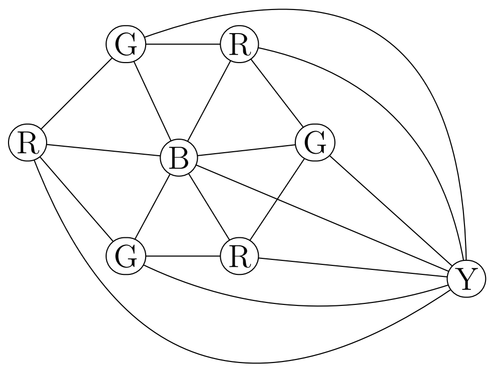
NETID:

FIRST: **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is four. The picture above shows how to color it with four colors (upper bound).

For the lower bound, the graph contains a W_6 whose hub is F and whose rim contains nodes A, B, C, D, E, H. Coloring a W_6 requires three colors. Then the node G is connected to all seven nodes in the W_6, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} \frac{1}{2^k}$ $\quad 1-(\frac{1}{2})^{n-1}$ ☐ $\quad 2-(\frac{1}{2})^{n}$ ☐ $\quad 1-(\frac{1}{2})^{n}$ ☑ $\quad 2-(\frac{1}{2})^{n-1}$ ☐

Putting 10 people in the canoe caused it to sink. 10 is _____ on how many people the canoe can carry.

an upper bound ☑ a lower bound ☐

neither ☐

The chromatic number of a graph with maximum vertex degree D

$= D$ ☐ $= D+1$ ☐

$\geq D+1$ ☐ $\leq D+1$ ☑

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1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} \; : \; |x+1| \leq 2\}$$
$$B = \{w \in \mathbb{R} \; : \; w^2 + 2w - 3 \leq 0\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution:

$A \subseteq B$: Let x be a real number and suppose $x \in A$. Then $|x+1| \leq 2$. Therefore, $-3 \leq x+1 \leq 1$. Therefore $x+3 \geq 0$ and $x-1 \leq 0$. So $x^2+2x-3 = (x+3)(x-1) \leq 0$. So $x \in B$.

$B \subseteq A$: Let x be a real number and suppose $x \in B$. Then $x^2+2x-3 \leq 0$. Factoring this polynomial, we get $(x+3)(x-1) \leq 0$. So $(x+3)$ and $(x-1)$ must have opposite signs. Since $x+3 > x-1$, it must be the case that $x+3 \geq 0$ and $x-1 \leq 0$. Therefore, $-3 \leq x+1 \leq 1$. So $|x+1| \leq 2$, and therefore $x \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=3}^{n} k^7 =$ $\sum_{p=1}^{n-2} p^9$ ☐ $\sum_{p=1}^{n-2} k^7$ ☐ $\sum_{p=1}^{n-2} k^9$ ☐ $\sum_{p=1}^{n-2} (p+2)^7$ ☑ $\surd$

The chromatic number of C_n. 2 ☐ 3 ☐ ≤ 3 ☑ $\surd$ ≤ 4 ☐

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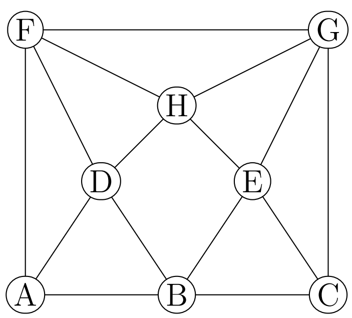
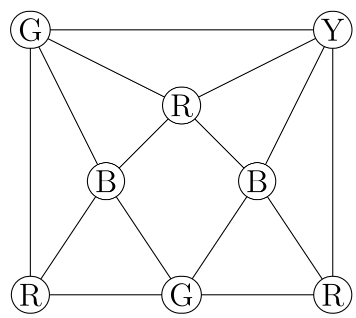
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





This graph has chromatic number four. The picture above shows that four colors are enough. If you delete node H (and all its edges), you get the special graph presented towards the end of section 10.3 in the textbook, which we know to require four colors.

Alternatively, you can directly argue the lower bound as follows. Suppose we try to color this with three colors. Color the triangle A, B, D with R, G, B respectively. Then F must have color G. Nodes E and C must have colors R and B in some order. But then node G has neighbors with all three colors, so we're stuck. Therefore three colors is not enough.

2. (6 points) Check the (single) box that best characterizes each item.

All elements of X are also elements of M. $M = X$ ☐ $M \subseteq X$ ☐ $X \subseteq M$ ☑ √

W_7 is a subgraph of G. 4 is ____ the chromatic number of G. exactly ☐ a lower bound on ☑ √ an upper bound on ☐

Chromatic number of G $\mathcal{C}(G)$ ☐ $\phi(G)$ ☐ $\chi(G)$ ☑ √ $\| G \|$ ☐

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1. (11 points) Let's define two sets as follows:

$$A = \{(p+1, p) \ : \ p \in \mathbb{R}\}$$
$$B = \{\lambda(1,0) + (1-\lambda)(2,1) \ : \ \lambda \in \mathbb{R}\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution:

$B \subseteq A$: Let (x, y) be a pair of real numbers such that $(x, y) \in B$. Then $(x, y) = \lambda(1,0) + (1-\lambda)(2,1)$ for some real number λ. Then $x = \lambda + 2 - 2\lambda = 2 - \lambda$ and $y = 1 - \lambda$. So $x = y + 1$. So (x, y) has the form $(p + 1, p)$ and therefore $(x, y) \in A$.

$A \subseteq B$: Let (x, y) be a pair of real numbers such that $(x, y) \in A$. Then $x = y+1$. Consider $\lambda = 1-y$. Then $y = 1-\lambda$ and $x = 2-\lambda = \lambda+2(1-\lambda)$. So $(x, y) = \lambda(1,0)+(1-\lambda)(2,1)$. Therefore $(x, y) \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

2. (4 points) Check the (single) box that best characterizes each item.

Suppose I want to estimate $\frac{103}{20}$. 3 is _____	an upper bound	☐	a lower bound	√	neither	☐		
Chromatic number of a bipartite graph with at least one edge	1	☐	2	√	3	☐	can't tell	☐

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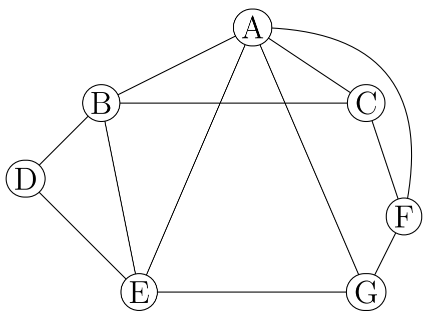
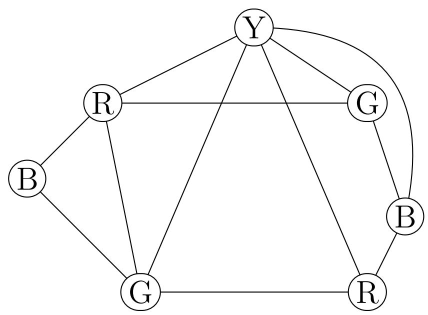
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is four. The picture above shows how to color it with four colors (upper bound). For the lower bound, the graph contains a W_5: the hub is node A and the rim contains nodes B, C, F, G, and E.

2. (6 points) Check the (single) box that best characterizes each item.

I found 143 identical marbles in my saucepan last Saturday. 143 is ____ how many marbles this size will fits in my saucepan.

exactly ☐ a lower bound on ☑ an upper bound on ☐

$\sum_{i=1}^{p-1} i =$ $\frac{p(p-1)}{2}$ ☑ $\frac{(p-1)^2}{2}$ ☐ $\frac{p(p+1)}{2}$ ☐ $\frac{(p-1)(p+1)}{2}$ ☐

The chromatic number of a graph with maximum vertex degree D

$= D$ ☐ $= D+1$ ☐ $\leq D+1$ ☑ $\geq D+1$ ☐

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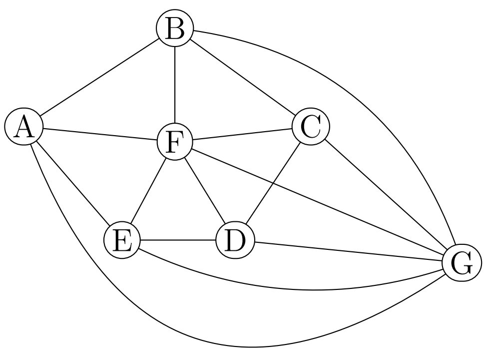
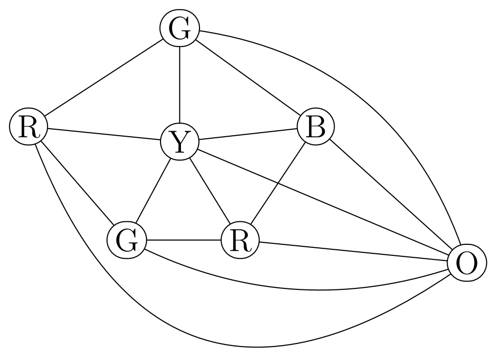
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors. Then the node G is connected to all six nodes in the W_5, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

Exactly 11 Xboxes fit in my suitcase by volume, but I haven't checked their total weight. 11 is _____ on how many Xboxes the suitcase can hold.	an upper bound ☑ $\surd$	a lower bound ☐	neither ☐

All elements of M are also elements of X.	$M = X$ ☐	$M \subseteq X$ ☑ $\surd$	$X \subseteq M$ ☐

$\sum_{k=1}^{n-1} \frac{1}{2^k}$	$1 - (\frac{1}{2})^n$ ☐	$2 - (\frac{1}{2})^n$ ☐	$1 - (\frac{1}{2})^{n-1}$ ☑ $\surd$	$2 - (\frac{1}{2})^{n-1}$ ☐