CS 173, Spring 2015
Examlet 9, Part A

NETID:

FIRST:

LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

(18 points) Here is a grammar G, with start symbol S and terminal symbols a and b.

$$\begin{aligned} S &\rightarrow a\ S\ a \mid S\ a\ S \mid a\ N\ a \\ N &\rightarrow a \mid b\ b \end{aligned}$$

Use (strong) induction to prove that any tree of height h matching (aka generated by) grammar G has at least h nodes with label a. Use $A(T)$ as shorthand for the number of a's in a tree T.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Let's define a Filbert tree to be a binary tree containing 2D points such that:

- Each leaf node contains $(3, 1)$, $(-2, -5)$, or $(2, 2)$.
- An internal node with one child labelled (a, b) has label $(a + 1, b - 1)$.
- An internal node with two childen labelled (x, y) and (a, b) has label $(x + a, y + b)$.

Use (strong) induction to prove that the point in the root node of any Filbert tree is on or below the line $x = y$.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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Recall that a full binary tree is one in which every node has either 0 or 2 children. An *Illini tree* is a full binary tree in which each node is colored orange or blue, such that:

- If v is a leaf node, then v may be colored orange or blue.
- If v has two children of the same color, then v is colored blue.
- If v has two children of different colors, then v is colored orange.

Use (strong) induction to show that the root of an Illini tree is blue if and only if the tree has an even number of orange leaves. You may assume basic divisibility facts e.g. the sum of two odd numbers is even.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Let's define an Interp Tree to be a binary tree containing 2D points such that:

- Each leaf node contains $(1, 2)$, $(5, 7)$, or $(-1, 10)$.
- An internal node with one child labelled (a, b) has label $(a, b + 1)$.
- An internal node with two childen labelled (x, y) and (a, b) has label $(\frac{x+a}{2}, \frac{y+b}{2})$.

Use (strong) induction to prove that thepoint in the root node of any Interp tree is above the line $x = y$

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) "Frumpy" trees are a type of tree whose nodes are labelled with integer values. Our recursive definition of frumpy trees states that a frumpy tree must be one of the following:

- A single node labelled 5 or 7.
- A tree with root labelled 0, with three frumpy trees as its children.
- A tree with root labelled 1, with two frumpy trees as its children.

The "total value" of a frumpy tree is the sum of the labels on all its nodes. Use (strong) induction that the total value of any frumpy tree is odd. You may assume basic divisibility facts e.g. the sum of two odd numbers is even.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Here is a grammar G, with start symbol S and terminal symbols a and p.

$$S \rightarrow S\ S \mid p\ S\ p \mid p\ p \mid a\ a$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar G has an even number of nodes with label p. Use $P(T)$ as shorthand for the number of p's in a tree T.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: