CS 173, Spring 2015
Examlet 9, Part B

NETID:

FIRST:

LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (8 points) Consider the following grammar G
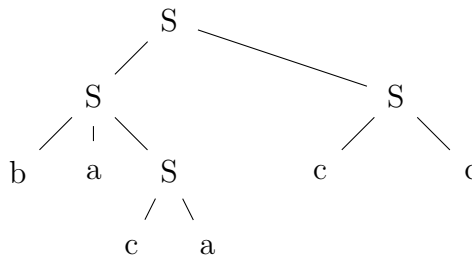
$$S \rightarrow b\,a\,S \mid S\,S \mid c\,c \mid c\,a$$

S is the only start symbol. The terminal symbols are a, b, and c.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

$c\,a\,b\,a\,c$ $b\,a\,c\,a\,c\,c$

This is impossible. The grammar produces terminals only in pairs and this string has odd length.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n+1} 2^k$$

$2^{n+1} + 1$ ☐ $2^{n+2} - 1$ ☑ $2^{n+2} - 2$ ☐ $2^{n+1} - 1$ ☐

A tree with n nodes has

n edges ☐ $n-1$ edges ☑ $\le n$ edges ☐

$n/2$ edges ☐ $\log n$ edges ☐

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1. (8 points) Consider the following grammar G

$$S \to S\,S \;\mid\; a\,N \;\mid\; a\,a$$
$$N \to S\,a \;\mid\; a\,b$$

S is the only start symbol. The terminal symbols are a and b

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

$b\ a\ a\ a\ b$

$a\ a\ a\ a\ b$

In the rules for grammar G, whenever b is produced, an a is produced right before it. So it's impossible to generate a string like this one which starts with a b.
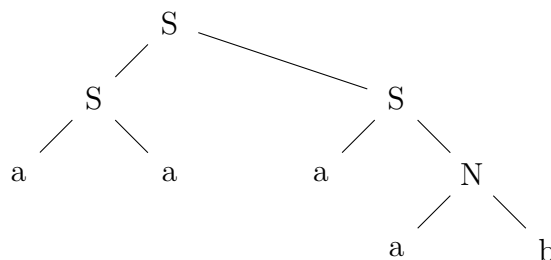
2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a full and complete 5-ary tree of height h	5^h ✓	$\le 5^h$ ☐		
	$\ge 5^h$ ☐	$5^{h+1} - 1$ ☐		

The chromatic number of a full 3-ary tree	1 ☐	2 ☐	≤ 2 ✓	
	3 ☐	≤ 3 ☐	can't tell ☐	

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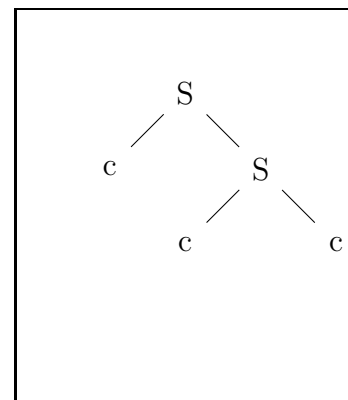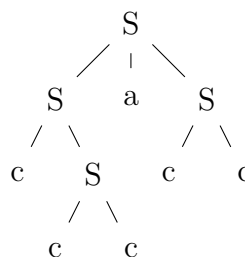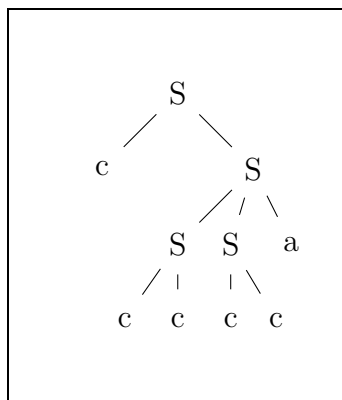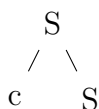**Examlet 9, Part B**

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Discussion: Monday   9   10   11   12   1   2   3   4   5

1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \to \; S\,S\,a \; | \; c\,S \; | \; c\,c$$

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^{n-1} 2^k$$

   $2^n + 1$ ☐ $2^n - 1$ ☐ $2^n - 2$ ☑ 2^n ☐

The root node of a tree is an internal node

   always ☐   sometimes ☑   never ☐

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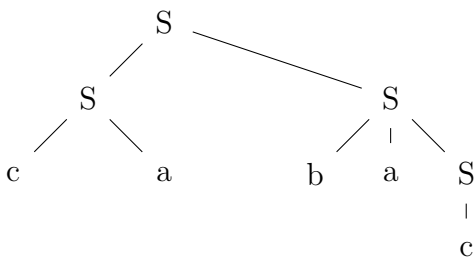
1. (8 points) Consider the following grammar G

   $$S \to b\,a\,S \mid S\,S \mid c \mid c\,a$$

   S is the only start symbol. The terminal symbol are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   c a b a c   *c a c b c*

In the rules for grammar G, every b is immediately followed by an a. So it's impossible to generate a string like this one which contains the sequence bc.

2. (4 points) Check the (single) box that best characterizes each item.

   $$\sum_{k=0}^{n-1} 2^k \qquad 2^n - 2 \;\square \qquad 2^n - 1 \;\boxed{\checkmark} \qquad 2^{n-1} - 1 \;\square \qquad 2^{n+1} - 1 \;\square$$

   A full m-ary tree with i internal nodes has $mi + 1$ nodes total.   always $\boxed{\checkmark}$ sometimes $\square$ never $\square$

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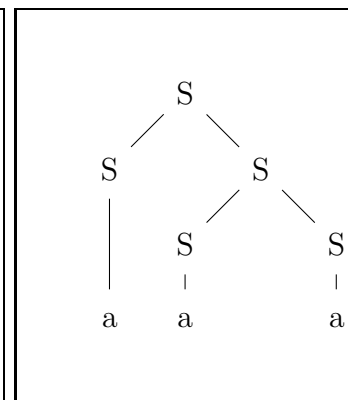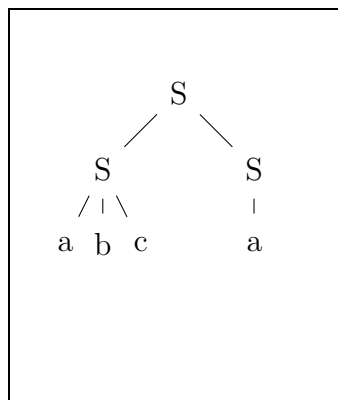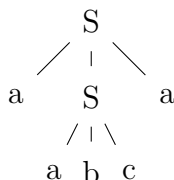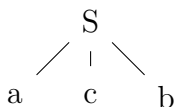**Examlet 9, Part B**

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Discussion: Monday   9   10   11   12   1   2   3   4   5

1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow \quad S\ S \ \mid \ a\ b\ c \ \mid \ a$$

2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h

$\geq 2^h$ ☐ $= 2^{h+1} - 1$ ☐

$\leq 2^{h+1} - 1$ ☑ $\geq 2^{h+1} - 1$ ☐

The level of the root node in a tree of height h.

0 ☑ 1 ☐

$h - 1$ ☐ h ☐ $h + 1$ ☐
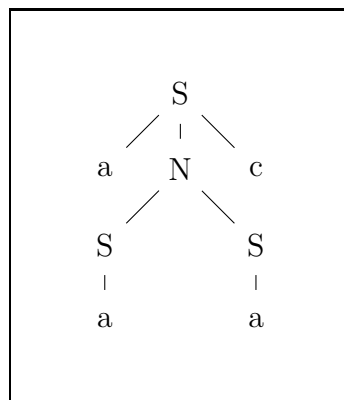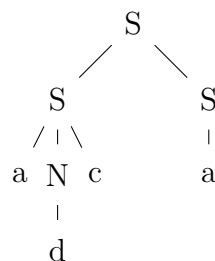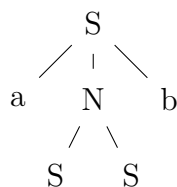
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LAST:

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow \ a \ N \ b \ | \ a \ N \ c \ | \ a$$
$$N \rightarrow \ S \ S \ | \ d$$

```
        S                    S                    S                        S
      / | \                / | \               /      \                 / | \
    b   N   a            a   N   b           S          S             a   N   c
        |                    / \           / | \        |               / \
        d                  S    S         a  N  c       a             S     S
                                             |                        |     |
                                             d                        a     a
```
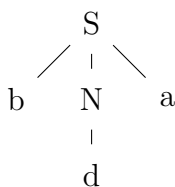
2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string $\emptyset$ ☐   e ☐ ϵ ☑   NULL ☐

$$\sum_{k=1}^{n+1} 2^k$$ $2^{n+1} + 1$ ☐ $2^{n+2} - 1$ ☐ $2^{n+2} - 2$ ☑ $2^n - 2$ ☐