CS 173, Spring 2015
Examlet 10, Part A

NETID:

FIRST:	**LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(15 points) Use (strong) induction to prove the following claim:

Claim: $n^2 < 2^n$ for any integer $n \geq 5$.

Hint: first prove that $2n + 1 \leq n^2$ for any integer $n \geq 5$. (This doesn't require induction.)

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{k=n+1}^{2n} \frac{1}{k} \geq \frac{7}{12}$, for any integer $n \geq 2$.

Hint: recall that if $x \leq y$, then $\frac{1}{y} \leq \frac{1}{x}$

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{N}$ be defined by

$f(1) = 0$

$f(n) = 1 + f(\lfloor n/2 \rfloor)$, for $n \geq 2$,

Use (strong) induction on n to prove that $f(n) \leq \log_2 n$ for any positive integer n. You cannot assume that n is a power of 2. However, you can assume that the log function is increasing (if $x \leq y$ then $\log x \leq \log y$) and that $\lfloor x \rfloor \leq x$.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1 + x)^n \geq 1 + nx$.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \leq 2\sqrt{n}$

Hint: notice that $(\sqrt{n} - \sqrt{n+1}\ )^2 \geq 0$. What does this imply about $2\sqrt{n}\sqrt{n+1}$?

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: