CS 173, Spring 2015
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (9 points) Fill in key facts about the recursion tree for T, assuming that T is even.

$$T(0) = 5 \qquad T(n) = 3T(n-2) + n^2$$

   (a) The height:
   $\frac{n}{2}$

   (b) The number of leaves (please simplify):
   $3^{\frac{n}{2}} = (\sqrt{3})^n$

   (c) Value in each node at level k:
   $(n-2k)^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

$(3^n)^2$ 10 $0.001n^3$ $30 \log n$ $n \log(n^7)$ $8n! + 18$ $3n^2$

10	$30 \log n$	$n \log(n^7)$	$3n^2$	$0.001n^3$	$(3^n)^2$	$8n! + 18$

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1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

   Solution:

   This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$?

no ☐ perhaps ☑ yes ☐

$n^{1.5}$ is

$\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☑

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

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Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

   $T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$

   (a) The height: $\log_7 n$.

   (b) The number of leaves (please simplify):
   $3^{\log_7 n} = 3^{\log_3 n \log_7 3} = n^{\log_7 3}$

   (c) Value in each node at level k:
   $(\frac{n}{7^k})^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2 \quad \frac{n \log n}{7} \quad (10^{10^{10}})n \quad 0.001n^3 \quad 30\log(n^{17}) \quad 8n! + 18 \quad 3^n + 11^n$

$30\log(n^{17})$	$(10^{10^{10}})n$	$\frac{n \log n}{7}$	$3n^2$	$0.001n^3$	$3^n + 11^n$	$8n! + 18$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

   Solution: There are positive reals c and k such that

   $0 \leq f(x) \leq cg(x)$

   for every $x \geq k$.

2. (8 points) Check the (single) box that best characterizes each item.

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

- $k < m$ ☑
- $k = m$ ☐
- $k > m$ ☐
- $km = 1$ ☐

$T(1) = d$
$T(n) = T(n/2) + c$

- $\Theta(\log n)$ ☑
- $\Theta(n)$ ☐
- $\Theta(n \log n)$ ☐
- $\Theta(n^2)$ ☐

3^n is

- $\Theta(5^n)$ ☐
- $O(5^n)$ ☑
- neither of these ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$?

- no ☐
- perhaps ☐
- yes ☑

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height:
   $\frac{n-1}{2}$

   (b) The number of leaves:
   $n(n-2)(n-4)\ldots 5 \cdot 3 \cdot 1$

   (c) Value in each node at level k:
   $n - 2k$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

$100 \log n$	$\log(2^n)$	$n \log(n^7)$	$(n^3)^7$	7^n	2^{3n}	$42n!$

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1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

   Solution: Yes, it is true. Suppose that $f(x)$ is $O(g(x))$. Then there are positive reals c and k such that $f(x) \leq cg(x)$ for all $x \geq k$. Then $\log(f(x)) \leq \log c + \log(g(x))$ for all $x \geq k$. Since $g(x)$ is an increasing function and $\log c$ isn't, There is some $K \geq k$ such that $\log c \leq \log(g(x))$. So then $\log(f(x)) \leq 2\log(g(x))$ for all $x \geq K$. So $\log(f(x))$ is $O(\log(g(x)))$.

   [You don't need this much technical detail for full credit. We can ignore the other inequality from the definition of big-O because the two functions always output positive values.]

2. (8 points) Check the (single) box that best characterizes each item.

   Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$?
   no ☐ perhaps ☐ yes ☑

   $T(1) = c$
   $T(n) = 3T(n/3) + n$
   $\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☑ $\Theta(2^n)$ ☐

   $n!$
   $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☑

   $n^{log_2 3}$ grows
   faster than n^2 ☐ slower than n^2 ☑
   at the same rate as n^2 ☐