CS 173, Spring 2015
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (9 points) Fill in key facts about the recursion tree for T, assuming that T is even.

   $T(0) = 5 \qquad T(n) = 3T(n-2) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(3^n)^2 \qquad 10 \qquad 0.001n^3 \qquad 30\log n \qquad n\log(n^7) \qquad 8n! + 18 \qquad 3n^2$

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Discussion: **Monday 9 10 11 12 1 2 3 4 5**

1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐
$\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$?

no ☐ perhaps ☐ yes ☐

$n^{1.5}$ is

$\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

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Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

   $T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2 \qquad \frac{n \log n}{7} \qquad (10^{10^{10}})n \qquad 0.001n^3 \qquad 30\log(n^{17}) \qquad 8n! + 18 \qquad 3^n + 11^n$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

2. (8 points) Check the (single) box that best characterizes each item.

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐

$\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

3^n is

$\Theta(5^n)$ ☐ $O(5^n)$ ☐ neither of these ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$?

no ☐ perhaps ☐ yes ☐

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Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height:

   (b) The number of leaves:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\quad 7^n$ $\quad 100\log n$ $\quad n\log(n^7)$ $\quad 2^{3n}$ $\quad \log(2^n)$ $\quad (n^3)^7$

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1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$?	no ☐	perhaps ☐	yes ☐
$T(1) = c$ $T(n) = 3T(n/3) + n$	$\Theta(n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐
$n!$	$O(2^n)$ ☐	$\Theta(2^n)$ ☐	neither of these ☐
$n^{log_2 3}$ grows	faster than n^2 ☐	slower than n^2 ☐	at the same rate as n^2 ☐