CS 173, Spring 2015
Examlet 12, Part A

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(a) (9 points) Suppose that I have a set of p nodes, labelled 1 through p. How many different graphs can I make with this fixed set of nodes? (Isomorphic graphs with differently labelled nodes count as different for this problem.) Briefly justify your answer.

Solution: There are $\frac{p(p-1)}{2}$ possible edges for the graph. For each one, we can choose to include it or not. So there are $2^{\frac{p(p-1)}{2}}$ different possible graphs.

(b) (6 points) State the negation of the following claim, in English (not shorthand notation), moving all negations (e.g. "not") so that they are on individual predicates.

> There is a bug b, such that for every plant p, if b pollinates p and p is showy, then p is poisonous.

Solution: For every bug b, there is a plant p, such that b pollinates p and p is showy, but p is not poisonous.

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(a) (9 points) Use proof by contradiction to show that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.

Solution: Suppose not. That is, suppose that there are positive integers x and y such that $x^2 - y^2 = 10$. Factoring the lefthand side, we get $(x-y)(x+y) = 10$. $(x-y)$ and $(x+y)$ must be integers since x and y are integers.

Ignoring sign, there are only two ways to factor 10: $2 \cdot 5$ or $1 \cdot 10$. In both cases, exactly one of the factors is odd, so the sum of the two factors is odd. But the sum of $(x-y)$ and $(x+y)$ is $2x$, which is even.

We have found a contradiction, so the original claim must have been correct.

(b) (6 points) Suppose a car dealer is planning to buy a set of Civics, Accords, and Fits (three kinds of cars). The dealer will buy ten cars in total and can buy any number of each type. How many different choices does he have? The sets are unordered, so three Civics and seven Fits is the same as seven Fits and three Civics.

Solution: Using the formula for combinations with repetition, there are

$$\binom{10+2}{2}$$

choices.

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(a) (9 points) A domino has two ends, each of which may be blank or contain between one and n spots. The two ends may have the same number of spots or different numbers of spots. A double-n domino set contains exactly one of each possible dot combination, where the order of the two ends doesn't matter. For example, a double-two domino set contains $(0, 0)$, $(1, 0)$, $(2, 0)$, $(1, 1)$, $(1, 2)$, and $(2, 2)$. Give a general formula for the number of dominoes in a double-n set, explaining why your formula is correct.

Solution: The double-n set contains $n + 1$ dominos with the same number of spots on both ends.

A domino with dissimilar ends corresponds to a set of two numbers in the range 0 through n. There are $\binom{n+1}{2} = \frac{n(n+1)}{2}$ such sets.

So, in total, there are $n + 1\frac{n(n+1)}{2} = \frac{(n+1)(n+2)}{2}$ dominoes in the set.

(b) (6 points) State the negation of the following claim, in English (not shorthand notation), moving all negations (e.g. "not") so that they are on individual predicates.

> For every computer game g, if g has trendy music or g has an interesting plotline, then g is not cheap.

Solution: There is a computer game g such that g has trendy music or an interesting plotline but g is cheap.

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(a) (9 points) Use proof by contradiction to show that, for any integer n, at least one of the three integers n, $2n + 1$, $4n + 3$ is not divisible by 7.

Solution: Suppose not. That is, suppose that n, $2n + 1$, $4n + 3$ are all divisible by 7. Then their sum $n + (2n + 1) + (4n + 3)$ must be divisible by 7. So $7n + 4$ must be divisible by 7. But then 4 would need to be divisible by 7, which isn't true.

Since its negation led to a contradiction, our original claim must have been true.

(b) (6 points) Chancellor Wise needs to construct a 13-person blue ribbon panel to find a new mascot, but she needs to decide how many members of the group are faculty, undergraduates, graduate students, and staff. How many ways can she choose the composition of the committee?

Solution: Using the formula for combinations with repetition, we get

$$\binom{13+3}{3}$$

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(a) (9 points) Suppose $A = \{a, b, c, d\}$. In how many different ways can A be partitioned? Briefly justify or show work.

Solution: The partition could contain anything from 1 to 4 subsets. Let's count these separately.

One subset: only one choice

Two subsets of two elements each: 3 choices (which other element is paired with a?)

A subset with one element and a subset with three elements: 4 choices

A subset with two elements and two subsets with one element: $\binom{4}{2} = 6$ choices

Four subsets: one choice

Total of 15 ways to build the partition.

(b) (6 points) State the negation of the following claim, in English (not shorthand notation), moving all negations (e.g. "not") so that they are on individual predicates.

For any student s, if s rides a bicycle, then s wears a helmet or s has no fear of death.

Solution: There is a student s who rides a bicycle but doesn't wear a helmet and fears death.

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(a) (9 points) Use proof by contradiction to show that, in any group of 7 people, there is at least one person who knows an even number of people. (Assume that "knowing someone" is symmetric.)

Solution: Suppose not. That is, suppose that we have a group of 7 people, in which each person knows an odd number of the other people.

Since knowing someone is symmetric, the total number of "knows" relationships must be even. (Each relationship goes both ways.) However, if we add up the number of supposed relationships, we have the sum of 7 odd numbers, which must be odd.

We have found a contradiction, so the original claim must have been correct.

(b) (6 points) In the polynomial $(3x - 2y)^{23}$, what is the coefficient of the term x^8y^{15}? (Please do not attempt to simplify your formula.)

Solution: According to the Binomial Theorem, this term is $\binom{23}{8}(3x)^8(-2y)^{15}$.

This is equal to $\binom{23}{8}3^8x^8(-2)^{15}y^{15}$.

So the coefficient on this term is $\binom{23}{8}3^8(-2)^{15}$.