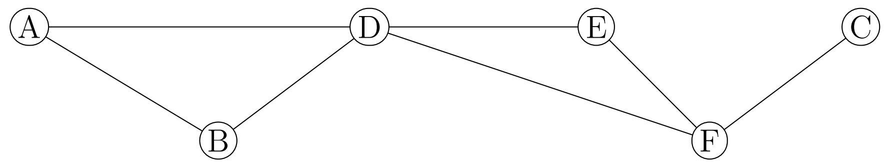
CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

Graph G is at right.
V is the set of nodes in G.



Define $f : V \to \mathbb{P}(V)$ by $f(p) = \{n \in V \ : \ \deg(n) \leq \deg(p)\}$, where deg(n) is the degree of node n.
Let $P = \{f(p) \mid p \in V\}$.

(a) (6 points) Fill in the following values:

f(A) = **Solution:** $\{A, B, C, E\}$

f(C) = **Solution:** $\{C\}$

P = **Solution:** $\{\{C\}, \{A, B, C, E\}, \{A, B, C, E, F\}, \}, \{A, B, C, D, E, F\}\}$

(b) (7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, it is not a partition. It doesn't contain the empty set (good), and it covers all of V (good), but there is partial overlap among its members (bad).

(c) (2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

Solution: $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST:	LAST:

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

Let $f : \mathbb{R} \to \mathbb{P}(\mathbb{R})$ be defined by $f(x) = \{p \in \mathbb{R} \mid \lfloor x \rfloor = \lfloor p \rfloor\}$
Let $T = \{f(x) \mid (x) \in \mathbb{R}\}$.

(a) (6 points) Answer the following questions:

$f(0) =$ **Solution:** $[0, 1) = \{x \in \mathbb{R} \ : \ 0 \leq x < 1\}$

Describe (at a high level) the elements of $f(7)$: **Solution:** All the real numbers whose floor is 7.

The cardinality of (aka the number of elements in) T is:

Solution: infinite

(b) (7 points) Is T a partition of $\mathbb{R}$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes. These intervals cover all of the real line, none of them is empty, and there is no partial overlap.

(c) (2 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 \ : \ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☑ infinite ☐

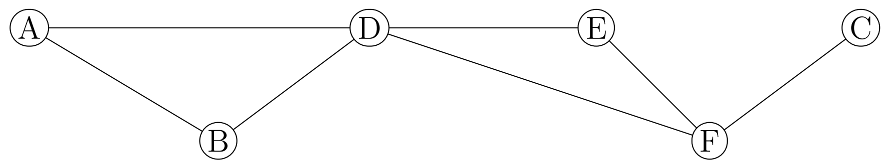
CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3, 4\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, F) = n\}$, where $d(a, b)$ is the distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(a) (6 points) Fill in the following values:

f(0) = **Solution:** $\{F\}$

f(1) = **Solution:** $\{C, D, E\}$

P = **Solution:** $\{\emptyset, \{F\}, \{C, D, E\}, \{A, B\}\}$

(b) (7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, P is not a partition of V. The subsets cover all of V with no partial overlap. However, P contains the empty set, since $f(3) = f(4) = \emptyset$.

(b) (2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ — true for all sets ☐ — true if $A \cap B = \emptyset$ ☐ — false for all sets ☑ ($\surd$)

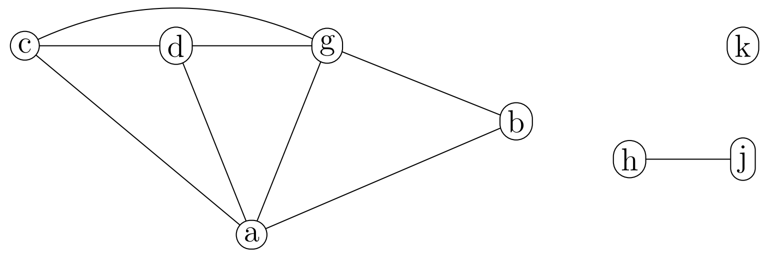
CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(a) (6 points) Fill in the following values:

$|V| =$ **Solution:** 8

$f(d) =$ **Solution:** $\{cd, ad, dg\}$

$f(h) =$ **Solution:** $\{hj\}$

(b) (7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, T is not a partition of E. T contains all edges in E. However, $f(k)$ is the empty set, so T contains the empty set. Also, there is partial overlap between the subsets, e.g. $f(d)$ and $f(a)$ are different but share the edge ad.

(c) (2 points) Check the (single) box that best characterizes each item.

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ undefined ☐

CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

(a) (9 points) Suppose that $A_1, A_2, \ldots, A_n$ are non-empty subsets of A, and let $P = \{A_1, A_2, \ldots, A_n\}$. Also suppose that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ and $A_1 \cup A_2 \cup \ldots \cup A_n = A$. Is P a partition of A? Explain why or why not.

Solution: P is not necessarily a partition of A. The issue is that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ can be true even when some pairs of (distinct) subsets overlap.

(b) (6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$

- true for all sets [√]
- true for some sets [ ]
- false for all sets [ ]

$\{\{a, b\}, c\} = \{a, b, c\}$

- True [ ]
- False [√]

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

- a rational [ ]
- a power set of rationals [ ]
- a set of rationals [√]
- undefined [ ]

CS 173, Spring 2015
Examlet 12, Part B

NETID:

FIRST:	**LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

Suppose that $A = \{2, 3, 5, 13, 17\}$. Let's define a function $F : A \to \mathbb{P}(A)$ and a set S as follows:

$$F(x) = \{y \in A \mid y \text{ is a factor of } x\}$$
$$S = \{F(x) \mid x \in A\}$$

(a) (2 points) List the members of $F(13)$. **Solution:** 13

(b) (7 points) Is S a partition of A? Why or why not?

Solution: Yes. S is a partition of A. Notice that $f(n) = \{n\}$ for all n in this particular set A. So element of A is in exactly one member of S and S cannot contain the empty set.

(c) (6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ — true for all sets ☐ true for some sets ☑ false for all sets ☐

$\mathbb{P}(\emptyset)$ — $\emptyset$ ☐ $\{\emptyset\}$ ☑ $\{\{\emptyset\}\}$ ☐ $\{\emptyset, \{\emptyset\}\}$ ☐

Pascal's identity states that $\binom{n}{k}$ is equal to — $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☑ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐