

CS 173, Spring 2015

Examlet 13, Part B

NETID:

FIRST:

LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

(5 points) Let's say that two graphs are distinct if and only if they are not isomorphic. Is the set of distinct (finite) planar graphs countable or uncountable? Briefly justify your answer.

Solution: This set is countable. If we fix a specific number of nodes, there are only finitely many distinct graphs with that many nodes. So the whole set is the union of countably many finite sets, which is countable.

(10 points) Check the (single) box that best characterizes each item.

The set of all finite-length strings of decimal digits

finite

☐

countably infinite

☒

uncountable

☐

$|A \times A| > |A|$

true

☐

false

☐

true for some sets

☒

Every function from the integers to the integers has a corresponding finite-length formula.

true

☐

false

☒

not known

☐

The set of all (finite) phone lattices using the 26 letters A, ..., Z.

finite

☐

countably infinite

☒

uncountable

☐

The set containing all functions f from the set of even integers to the set of even integers

finite

☐

countably infinite

☐

uncountable

☒

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(5 points) A “pretty wheel” graph is a wheel graph whose vertices are colored with colors from the set {red, green, blue, violet, yellow}. Two pretty wheels are distinct if they have a different number of nodes or if they have the same number of nodes but a different color pattern. Is the set of distinct pretty wheels countable or uncountable? Briefly justify your answer.

Solution: This is countable. For any n , there are only finitely many ways to color the nodes of W_n . So the whole set is the union of countably many finite sets, which is countable.

(10 points) Check the (single) box that best characterizes each item.

There are mathematical functions that don't have a finite formula.

true

☒

false

☐

not known

☐

The rational numbers have the same cardinality as the reals.

true

☐

false

☒

not known

☐

The set of all finite sequences of Chinese characters.

finite

☐

countably infinite

☒

uncountable

☐

If $f : A \rightarrow B$ is one-to one

$|A| < |B|$

☐

$|A| \leq |B|$

☒

$|A| = |B|$

☐

$\mathbb{P}(\mathbb{N})$

finite

☐

countably infinite

☐

uncountable

☒