CS 173, Fall 2016
Examlet 2, Part A

NETID:

FIRST:	LAST:

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers p and q ($p \neq -1$), if $\frac{2}{p+1}$ and $p+q$ are rational, then q is rational.

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(15 points) For any two real numbers x and y, the harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Using this definition and your best mathematical style, prove the following claim:

For any real numbers x and y, if $0 < x < y$, then $H(x, y) < y$.

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(15 points) Prove the following claim, using your best mathematical style and the following definition of
congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers a, b, c, p and k (c positive), if $ap \equiv b \pmod{c}$ and $k \mid a$ and $k \mid c$, then $k \mid b$.

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(15 points) For any two real numbers x and y, the Harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Use this definition and your best mathematical style to prove the following claim:

For all positive real numbers x and y and p, if $x \geq y$, then $H(x, p) \geq H(y, p)$.

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(15 points) Prove the following claim, using your best mathematical style and the following definition of
congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers x, y, p, q and m, with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $x^2 + xy \equiv p^2 + pq \pmod{m}$.

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(15 points) The following claim is slightly buggy. Explain briefly what values cause it to fail, add one extra (very simple) condition to the hypothesis. Then prove your revised claim, working directly from the definition of "divides" and using your best mathematical style.

For any integers a, b, and c, if $a^2b \mid cb$, then $a \mid c$.