CS 173, Fall 2016
Examlet 2, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7839, 1474)$. Show your work.

   Solution:

   $7839 - 5 \times 1474 = 7839 - 7370 = 469$

   $1474 - 3 \times 469 = 1474 - 1407 = 67$

   $469 - 7 \times 67 = 0$

   So the GCD is 67.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod 5$ true [√] false [ ]

   $\gcd(p,q) = \frac{pq}{\text{lcm}(p,q)}$ (p and q positive integers) always [√] sometimes [ ] never [ ]

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 2 \pmod 6$ and $n \equiv 1 \pmod 9$?

   Solution: There is no such n. If $n \equiv 2 \pmod 6$ and $n \equiv 1 \pmod 9$, then $n = 2 + 6k$ and $n = 1 + 9j$, where k and j are integers. So $2 + 6k = 1 + 9j$. This implies that $1 = 9j - 6k$ which is impossible because the right side is divisible by 3 and the left side isn't.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution: $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So $\gcd(1702, 1221) = 37$

3. (4 points) Check the (single) box that best characterizes each item.

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false ☑ ($\surd$)

   $25 \equiv 4 \pmod 7$ true ☑ ($\surd$) false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is false. Consider $a = 3$ and $b = -3$. Then $a \mid b$ and $b \mid a$, but $a \neq b$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(221, 1224)$. Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. true [√] false [ ]

   $-7 \mid 0$ true [√] false [ ]

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 2 \pmod 6$ and $n \equiv 5 \pmod{15}$?

   Solution: This is true. Consider $n = 20$. $20 \equiv 2 \pmod 6$ and $20 \equiv 5 \pmod{15}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(221, 1224)$. Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(a, b) = \gcd(r, a)$ — true ☐ false ☑ ($\surd$)

   $7 \equiv -7 \pmod k$ — always ☐ sometimes ☑ ($\surd$) never ☐

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid c$ and $b \mid c$, then $ab \mid c$

   Solution: This is not true. Consider $a = b = c = 5$. Then $a \mid c$ and $b \mid c$. But $ab = 25$ and $c = 5$. So ab does not divide c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(535, 1819)$. Show your work.

   Solution:

   $535 - 0 \times 1819 = 535$

   $1819 - 3 \times 535 = 1819 - 1605 = 214$

   $535 - 2 \times 214 = 535 - 428 = 107$

   $214 - 2 \times 107 = 0$

   So the GCD is 107.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(r, a)$ — true ☐ false ☑

   $-7 \equiv 13 \pmod 5$ — true ☑ false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ and $\gcd(a, c) > 1$.

   Solution: This is false. Consider $a = b = 3$ and $c = 2$. Then $bc = 6$. So $\gcd(a, bc) = 3 > 1$ but $\gcd(a, c) = 1$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2385, 636)$. Show your work.

   Solution:

   $2385 - 3 \times 636 = 2385 - 1908 = 477$

   $636 - 477 = 159$

   $477 - 3 \times 159 = 0$

   So the GCD is 159.

3. (4 points) Check the (single) box that best characterizes each item.

   If p, q, and k are positive integers, then $gcd(pq, qk) =$ q ☐ pq ☐ pqk ☐ $q\gcd(p, k)$ ☑

   $0 \mid 7$ true ☐ false ☑