CS 173, Fall 2016
Examlet 3, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (4 points) $M = \{\text{cereal}, \text{toast}\}$ $N = \{\text{milk}, \text{coffee}, \text{wine}\}$
   $P = \{\text{wine}, \text{beer}, (\text{coffee}, \text{ham}), (\text{milk}, \text{ham})\}$

   Solution:

   $M \times (N - P) = M \times \{\text{milk}, \text{coffee}\} = \{(\text{cereal}, \text{milk}), (\text{cereal}, \text{coffee}), (\text{toast}, \text{milk}), (\text{toast}, \text{coffee})\}$

   $|M \times N \times P| = 2 \cdot 3 \cdot 4 = 24$

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{R}$, if $x^2 = 3$, then $x > 1000$. true ☐ false ☑ undefined ☐

   $A \cap (B \cup C)$ $= (A \cap B) \cup (A \cap C)$ true for all sets A ☑ true for some sets A ☐ false for all sets A ☐

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{42} = [5]^{32} \cdot [5]^8 \cdot [5]^2 = [1][-1][8] = [-8] = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   For all positive integers n,
   if $n! < -10$, then $n > 8$. true [√] false [ ] undefined [ ]

   $A \times B = A$ true for all sets A and B [ ] false for all sets A and B [ ]
   true for some sets A and B [√]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{38} = [5]^{32} \cdot [5]^4 \cdot [5]^2 = [7] \cdot [4] \cdot [7] = [28] \cdot [7] = [1] \cdot [7] = [7]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \cup (B - C) \subseteq (A \cup B) - C$

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{2, 3\}$.
   Then $A \cup (B - C) = \{1, 2\} \cup \{4\} = \{1, 2, 4\}$.
   But $(A \cup B) - C = \{1, 2, 3, 4\} - \{2, 3\} = \{1, 4\}$.
   So $A \cup (B - C) \not\subseteq (A \cup B) - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   Sets A and B are disjoint
   $A - B = B - A$ ☐ $A = \overline{B}$ ☐
   $A \cap B = \{\emptyset\}$ ☐ $A \cap B = \emptyset$ ☑

   $\emptyset$ is
   an element of $\mathbb{Z}$ ☐ a subset of $\mathbb{Z}$ ☑ both ☐ neither ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:
   $[7]^2 = [49] = [10]$
   $[7]^4 = [100] = [9]$
   $[7]^8 = [9]^2 = [81] = [3]$
   $[7]^{16} = [3]^2 = [9]$
   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9] \cdot [10] \cdot [7]$
   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$
   So $[7]^{19} = [6]$

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1. (4 points) $A = \{\text{fox, tiger, wolf, eagle, cat}\}$ $B = \{3, 4\}$ $C = \{6, 7\}$

   Solution:

   $A \times (B \cap C) = A \times \emptyset = \emptyset$

   $| A \times (B \cup C) | = 4 \times 5 = 20$

2. (4 points) Check the (single) box that best characterizes each item.

   Sets A and B are disjoint
   $A \cap B = \{\emptyset\}$ ☐ $A \cap B = \emptyset$ ☑
   $|A \cap B| = 1$ ☐ $A = \overline{B}$ ☐

   $\emptyset \times \emptyset =$
   $\emptyset$ ☑ $\{\emptyset\}$ ☐ $\{\emptyset, \emptyset\}$ ☐ $\{(\emptyset, \emptyset)\}$ ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{40}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{40} = [7]^{32} \cdot [7]^8 = [5] \cdot [-2] = [-10] = [1]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A \cup B) - C = A \cup (B - C)$.

   Solution:

   Let $A = \{1, 2\}$, $B = \{3\}$, and $C = \{2\}$.

   Then $(A \cup B) - C = \{1, 2, 3\} - C = \{1, 3\}$.

   But $A \cup (B - C) = \{1, 2\} \cup \{3\} = \{1, 2, 3\}$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cup B$, then $x \in A$.

   true for all sets A and B ☐ true for some sets A and B ☑

   false for all sets A and B ☐

   $\{1, 2\} \cup \emptyset =$

$\emptyset$	☐	$\{(1, \emptyset), (2, \emptyset)\}$	☐	$\{1, 2, \emptyset\}$	☐
$\{\emptyset\}$	☐	$\{1, 2\}$	☑	undefined	☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$

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1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$ $C = \{3, 7\}$

   Solution:

   $A \times (B \cap C) = A \times \{3\} = \{(\text{fox}, 3), (\text{cat}, 3)\}$

   $A \cap B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| = |A| + |B|$ — true for all sets A [ ] true for some sets A [√] false for all sets A [ ]

   $\emptyset \times A = A \times \emptyset$ — true for all sets A [√] false for all sets A [ ] true for some sets A [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{32} = [3]^2 = [9]$

   $[8]^{37} = [8]^{32} \cdot [8]^4 \cdot [8] = [9] \cdot [4] \cdot [8] = [36] \cdot [8] = [3] \cdot [8] = [24] = [2]$