CS 173, Fall 2016
Examlet 4, Part A

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Suppose that n is some positive integer. Let's define the relation R_n on the integers such that aR_nb if and only if $a \equiv b+1 \pmod{n}$. Prove the following claim

Claim: For any integers x, y, and z, if xR_ny and yR_nz and xR_nz, then $n = 1$.

You must work directly from the definition of congruence mod k, using the following version of the definition: $x \equiv y \pmod{k}$ iff $x - y = mk$ for some integer m. You may use the following fact about divisibility: for any non-zero integers p and q, if $p \mid q$, then $|p| \leq |q|$.

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Let's define a relation T between pairs of natural numbers as follows:

$(a, b)T(p, q)$ if and only if $a \mid p$ and $b = q$.

Working directly from this definition and the definition of divides, prove that T is antisymmetric.

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Let T be the relation defined on $\mathbb{N}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or $(x = p$ and $y \leq q)$

Prove that T is transitive.

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Recall how to multiply a real number α by a 2D point $(x, y) \in \mathbb{R}^2$: $\alpha(x, y) = (\alpha x, \alpha y)$.

Let $A = \mathbb{R}^+ \times \mathbb{R}^+$, i.e. pairs of positive real numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists a real number $\alpha \geq 1$ such that $(x, y) = \alpha(p, q)$.

Prove that $\gg$ is antisymmetric.

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Let's define a relation T between natural numbers follows:

aTb if and only if $a = b + 2k$ where $k \in \mathbb{N}$

Working directly from this definition, prove that T is antisymmetric.

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) = (pq)(x + y)$

Prove that T is transitive.