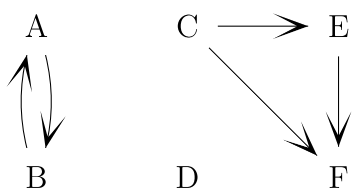
CS 173, Fall 2016
Examlet 4, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: √

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(x, y) \sim (p, q)$ if and only $|x| + |y| = |p| + |q|$. List three members of $[(2, 3)]$.

   Solution: (2,3), (-2,3), (1, -4)

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $xy = 1$ for all integers x and y. Is R a partial order?

   Solution: No, R is not a partial order. Notice that the only relations are when $x = y = 1$ or $x = y = -1$. So it's transitive and symmetric, but not reflexive.

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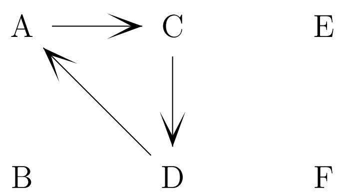

Reflexive:	☐	Irreflexive:	$\surd$
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	☐		

2. (5 points) Can a relation be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. However, this must be the empty relation in which no elements are related. (No edges in the graph representation.) If we have aRb, then we must have bRa (by symmetry) and so aRa (by transitivity), which is inconsistent with the relation being irreflexive.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. We have $(0, 0)T(-10, 10)$ (look at the second coordinate). We also have $(-10, 10)T(-5, -5)$ (look at the first coordinate). But it's not the case that $(0, 0)T(-5, -5)$.

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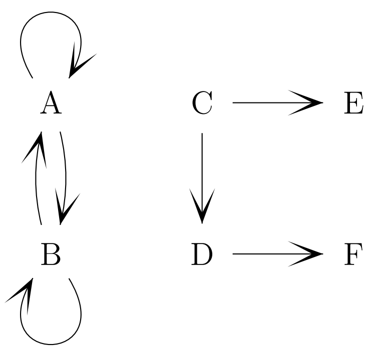
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) A relation is a strict partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: irreflexive, antisymmetric, transitive

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $\gcd(a, b) > 1$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution:

   This relation is not transitive. Consider 2, 6, and 3. Then $\gcd(2, 6) > 1$ and $\gcd(6, 3) > 1$, but $\gcd(2, 3) = 1$.

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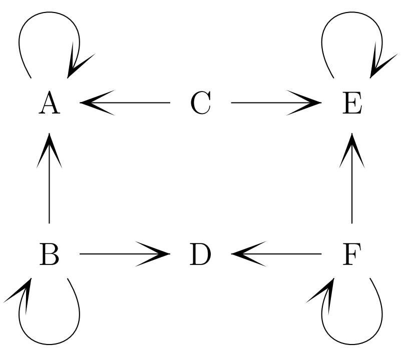

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ √

Transitive: ☑ √

2. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(a, b) \sim (p, q)$ if and only $aq = bp$. List three members of $[(5, 6)]$.

   Solution: (5,6), (10,12), (-5, -6)

3. (5 points) Let S be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)S(p, q)$ if and only if $x^2 + y^2 \leq p^2 + q^2$. Is S antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not antisymmetric. We have $(0, 1)S(1, 0)$ and $(1, 0)S(0, 1)$, but $(0, 1) \neq (1, 0)$.

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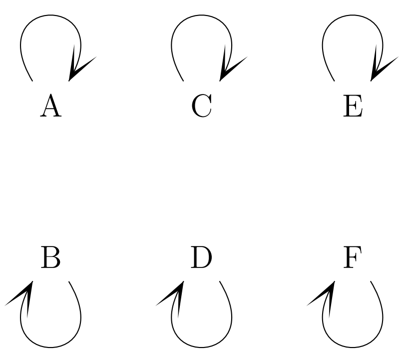
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	√	Irreflexive:	
Symmetric:	√	Antisymmetric:	√
Transitive:	√		

2. (5 points) Let R be the relation on the integers such that xRy if and only if $\lfloor x/4 \rfloor = \lfloor y/4 \rfloor$. List the values in [8].

   Solution: [8] contains (only) 8, 9, 10, and 11

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $2 \mid (x + y + 1)$. Is R transitive?x

   Solution: No, R is not transitive. For example, $2R3$ and $3R4$ but it's not the case that $2R4$.

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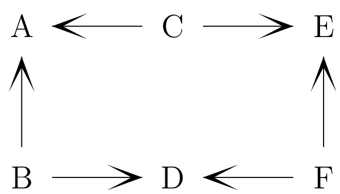
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☑ √
Symmetric:	☐	Antisymmetric:	☑ √
Transitive:	☑ √		

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lceil x \rceil = \lceil y \rceil$. Give three members of the equivalence class [13].

   Solution: 13, 12.5, 12.3

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "disjoint" relation D on J by $(a, b)D(c, d)$ if and only if $b \leq c$ or $d \leq a$. Is D transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: D is not transitive. Consider $(1, 2)$, $(3, 5)$, and $(4, 6)$. Then $(4, 6)D(1, 2)$. $(1, 2)D(3, 5)$ and But not $(4, 6)D(3, 5)$.