CS 173, Fall 2016
Examlet 5, Part A

NETID:

FIRST:	LAST:

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (2f(x) + f(y), f(x) - f(y))$. Prove that g is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) $A = \{1, 3, 5, 7, 9, \ldots\}$, i.e. the positive odd numbers.

   $B = \{-1, -2, -3, -4, -5 \ldots\}$, i.e. negative numbers

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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1. (10 points) Suppose that $g : \mathbb{N} \to \mathbb{N}$ is one-to-one. Let's define the function $f : \mathbb{N}^2 \to \mathbb{N}^2$ by the equation $f(x, y) = (x + g(y), g(x))$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

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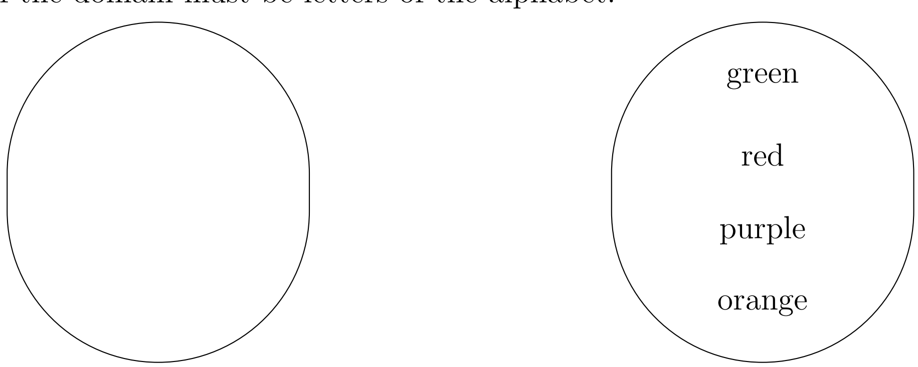
1. (10 points) Suppose that $f : [0, \frac{1}{2}] \rightarrow [1, \frac{5}{2}]$ is defined by $f(x) = \frac{x^2+1}{1-2x^2}$ Prove that g is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Complete this picture to make an example of a function that is onto but not one-to-one, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.



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1. (10 points) Suppose that $g : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $h : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $h(x, y) = (g(x) + g(y), g(x) - g(y))$. Prove that h is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is onto but not one-to-one. Be specific.

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1. (10 points) Suppose that $f : \mathbb{Z}^2 \to \mathbb{Z}$ is defined by $f(x, y) = xy + yx^2 - x^2$. Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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1. (10 points) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that f is onto.

2. (5 points) Suppose that $g : A \to B$ and $f : B \to C$. Prof. Snape claims that if $f \circ g$ is one-to-one, then f is one-to-one. Disprove this claim using a concrete counter-example in which A, B, and C are all small finite sets.