CS 173, Fall 2016
Examlet 5, Part B

NETID:

FIRST: **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (5 points) Suppose that $|A| = 2$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: There are no onto functions from A to B, because $|A|$ is smaller than $|B|$.

2. (10 points) Check the (single) box that best characterizes each item.

   A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one. true ☐ false ☑

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ onto ☑ not onto ☐ not a function ☐

   $g : (0, \frac{\pi}{2}) \to \mathbb{R}$, $g(x) = \sin(x)$ one-to-one ☑ not one-to-one ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have the same gift. true ☑ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$ true ☑ false ☐

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1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, and shocking pink. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

   Solution: She needs to pull out four socks. By the pigeonhole principle, four socks and only three colors means that two must have the same color.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is onto if and only if its image is the same as its co-domain. true [√] false [ ]

   $f : \mathbb{Z} \to \mathbb{Z}$,
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd)
   one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{R} \to \mathbb{R}$,
   $g(x) = \sin(x)$
   onto [ ] not onto [√] not a function [ ]

   $g : \mathbb{R} \to \mathbb{Z}$,
   $g(x) = \lfloor x \rfloor$
   one-to-one [ ] not one-to-one [√] not a function [ ]

   $\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)
   true [√] false [ ]

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1. (5 points) How many different 9-letter strings can be made be rearranging the characters in the word ‘‘`silliness`’’? Show your work.

   Solution: There are 9 letters total, with 3 copies of s, two l's, and 2 i's. So the number of possibilities is

   $$\frac{9!!}{3!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If a function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing, it must be one-to-one. true [√] false [ ]

   $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ (x even), $f(x) = x - 21$ (x odd) onto [√] not onto [ ] not a function [ ]

   $f : \mathbb{N}^2 \to \mathbb{Z}$, $f(p, q) = 2^p 4^q$ one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$ one-to-one [√] not one-to-one [ ] not a function [ ]

   $\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$ ($\mathbb{R}^+$ is the positive real numbers.) true [ ] false [√]

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1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

   Solution: q^p

2. (10 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has at least one pre-image. true [√] false [ ]

   $g : \mathbb{R} \to \mathbb{R}$, $g(x) = \sin(x)$ one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$ onto [ ] not onto [√] not a function [ ]

   $f : \mathbb{N} \to \mathbb{N}$, $f(x) = 3 - x$ one-to-one [ ] not one-to-one [ ] not a function [√]

   $\forall p \in \mathbb{Z}^+,\ \exists t \in \mathbb{Z}^+,\ \gcd(p, t) = 1$ true [√] false [ ]

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1. (5 points) Prof. Snape is teaching potions to 52 girls and 73 boys. Quiz 1 has integer scores between zero and 100. Assuming no one missed the quiz, what is the probability that two students got the same score? Briefly justify your answer.

   Solution: There are 101 distinct quiz scores and 125 students. By the pigeonhole principle, two students got the same score. So probability 1.0 (or 100% if you prefer that notation).

2. (10 points) Check the (single) box that best characterizes each item.

   If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false ☑

   $f : \mathbb{Z} \to \mathbb{R}$, $f(x) = x$ one-to-one ☑ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$ one-to-one ☑ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$ onto ☐ not onto ☑ not a function ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$ true ☐ false ☑

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 2$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: It doesn't matter what the elements of A and B are, so let's suppose that $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Two elements of A must map to the same output value, with the third element x mapping to the other output value. There are three choices for which element x is. And then there are two choices for which output value corresponds to x. So 6 onto functions total.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the ____ of f.
   domain ☐ co-domain ☑ image ☐

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = |x|$
   one-to-one ☑ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{N}$, $g(x) = |x|$
   onto ☑ not onto ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm.
   true ☐ false ☑

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$
   true ☑ false ☐