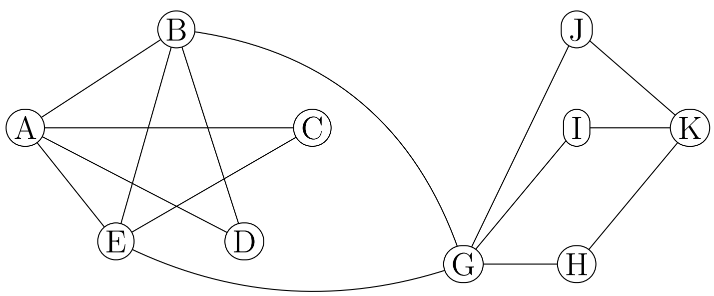
CS 173, Fall 16
Examlet 6, Part A

NETID:

FIRST: | **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes A, G, and K must map onto themselves.

B and E can swap (2 choices). This fixes C and D. I, J, and H can be permuted (6 choices). So there are a total of $2 \cdot 6 = 12$ choices.

2. (5 points) Is C_5 a subgraph of W_7? Briefly justify your answer.

Solution: Yes. The copy of C_5 consists of 4 consecutive nodes along the rim, plus the hub of the wheel. (A picture would also work here.)

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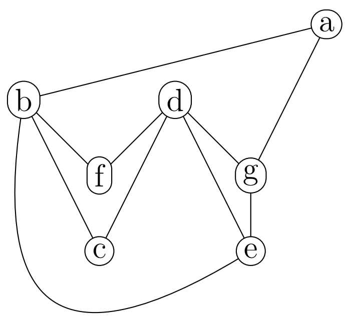
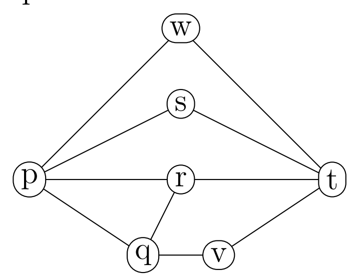
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: Yes. We can map the nodes as follows:

f(b) = t, f(d) = p, f(f) = w, f(c) = s,, f(e) = r, f(g) = q), f(a) = v.

2. (5 points) Use the pigeonhole principle to briefly explain why a graph with n nodes ($n \geq 2$) must have two nodes with the same degree. Hint: if one node has degree 0, what is the maximum degree of any other node?

Solution: Node degrees range from 0 to $n-1$. However, it's impossible to have both a node with degree 0 and a node with degree $n-1$. So the degrees in any specific graph must either be between 0 and $n-2$, or else between 1 and $n-1$. In both cases, there are only $n-1$ distinct degrees. But there are n nodes. So two nodes must have the same degree.

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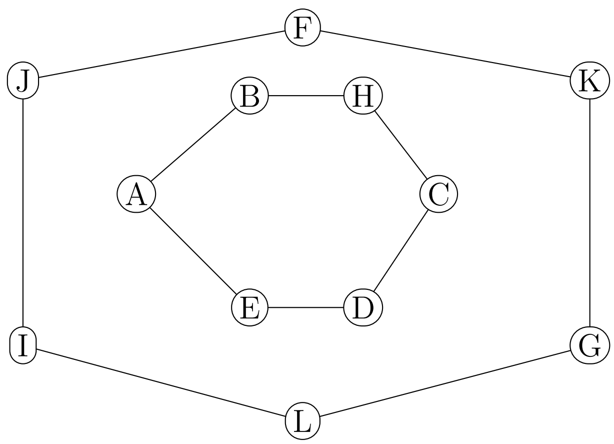

Solution: F can match any of the 12 nodes. K must match to one of the two neighbors of F's match. This determines the match for G, I, J, L.

B can then match to any of the 6 nodes on the other right from F's match. This leaves two choices for H. And then this fixes the match for A, C, D, E.

So the total number of choices is $12 \cdot 2 \cdot 6 \cdot 2 = 288$.

2. (5 points) The complete graph K_7 contains 7 nodes. How many edges does it have?

Solution: It has $\frac{7 \cdot 6}{2} = 21$ edges.

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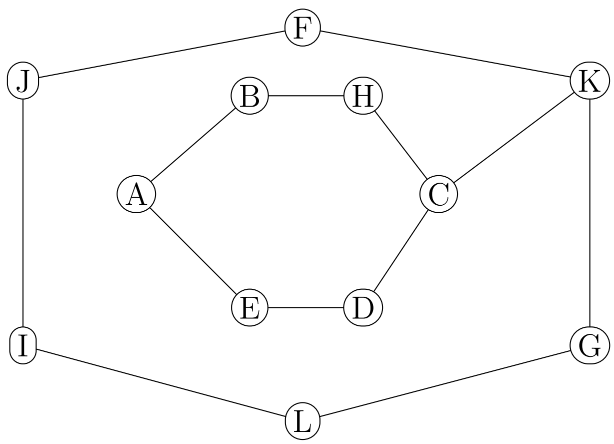

Solution: C and K can swap (2 choices). Then D and H can swap (2 choices). And F and G can swap (2 choices). So there are a total of 8 choices.

2. (5 points) How many edges are in the complete bipartite graph $K_{11,6}$?

Solution: $11 \times 6 = 66$

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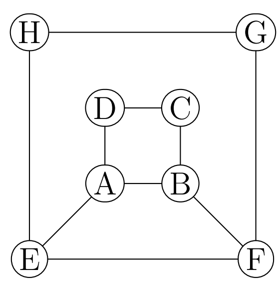
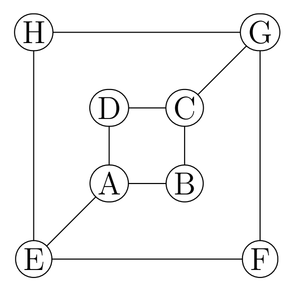
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: No, they are not isomorphic. Graph X has adjacent pairs of degree-2 nodes (e.g. C and D). Graph Y has no such pair.

2. (5 points) If G is a graph, its complement G' has the same nodes as G but G' has an edge between nodes x and y if and only G does not have an edge between x and y. Give a succinct high-level description of the complement of C_5. Briefly justify or show work.

Solution: Suppose we label the nodes of C_5 (in order) as a, b, c, d, and e. In the complement, we have edges ac, ad, bd, be, and ce. Rearranging them gives us: ac, ce, eb, bd, da. So the complement is also a 5-cycle.

[A picture would also be a good way to justify your answer.]

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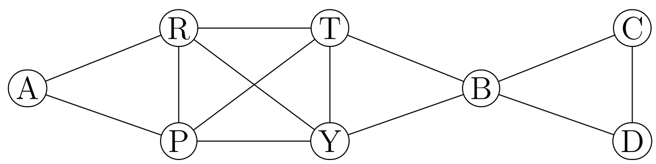
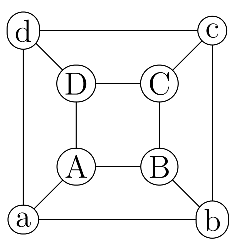


Solution: A and B must map onto themselves. R and P can swap (2 choices) T and Y can swap independently of R and P (2 choices). And C can also swap with D. So there are 8 choices total.

2. (5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. Put nodes a, c, B, and D into one set and nodes A, C, b, and d into the other set.