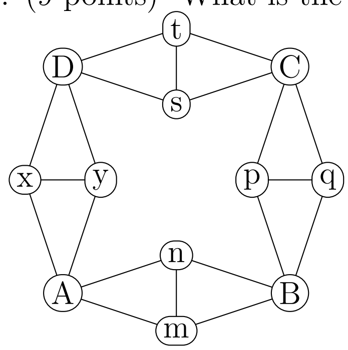
CS 173, Fall 2016
Examlet 7, Part B

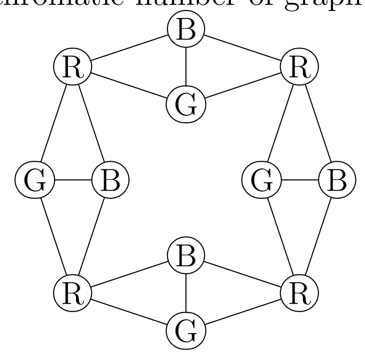
NETID:

FIRST: | **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is three. The picture above shows that it can be colored with three colors (upper bound). Since it contains triangles, we also have a lower bound of three.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p-1} i$ — $\frac{(p-1)^2}{2}$ ☐ $\frac{(p-1)(p+1)}{2}$ ☐ $\frac{p(p+1)}{2}$ ☐ $\frac{p(p-1)}{2}$ [√]

Leal team's bridge collapsed under a 100 pound weight. 100 pounds is ______ on how much the bridge can hold.

- an upper bound on [√]
- exactly ☐
- a lower bound on ☐
- not a bound on ☐

Chromatic number of a bipartite graph with at least one edge — 1 ☐ 2 [√] 3 ☐ can't tell ☐

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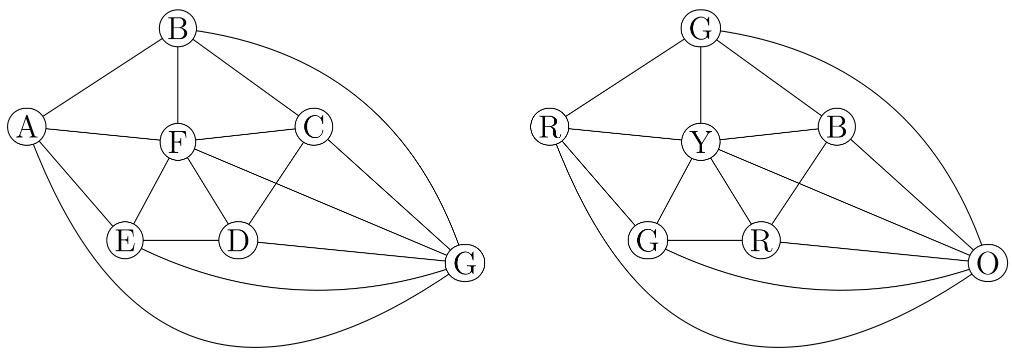

Solution: The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors. Then the node G is connected to all six nodes in the W_5, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n-1} \frac{1}{2^k}$ $1-(\frac{1}{2})^n$ ☐ $2-(\frac{1}{2})^n$ ☐ $1-(\frac{1}{2})^{n-1}$ ☑ $2-(\frac{1}{2})^{n-1}$ ☐

$\pi \leq 7.3$ an upper bound on π ☑ exactly π ☐ a lower bound on π ☐ not a bound on π ☐

Chromatic number of a graph with maximum vertex degree D $= D$ ☐ $= D+1$ ☐ $\geq D+1$ ☐ $\leq D+1$ ☑

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1. (11 points) Let's define two sets as follows:

$$A = \{(p+1, p) \ : \ p \in \mathbb{R}\}$$
$$B = \{\lambda(1,0) + (1-\lambda)(2,1) \ : \ \lambda \in \mathbb{R}\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution: $B \subseteq A$: Let (x, y) be a pair of real numbers such that $(x, y) \in B$. Then $(x, y) = \lambda(1,0) + (1-\lambda)(2,1)$ for some real number λ. Then $x = \lambda + 2 - 2\lambda = 2 - \lambda$ and $y = 1 - \lambda$. So $x = y + 1$. So (x, y) has the form $(p+1, p)$ and therefore $(x, y) \in A$.

$A \subseteq B$: Let (x, y) be a pair of real numbers such that $(x, y) \in A$. Then $x = y + 1$. Consider $\lambda = 1 - y$. Then $y = 1 - \lambda$ and $x = 2 - \lambda = \lambda + 2(1-\lambda)$. So $(x, y) = \lambda(1,0) + (1-\lambda)(2,1)$. Therefore $(x, y) \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with no edges	1 [√]	2 [ ]	3 [ ]	can't tell [ ]

Suppose I want to estimate $\frac{103}{50}$. 3 is _____

an upper bound [√]	an exact answer [ ]
a lower bound [ ]	not a bound on [ ]

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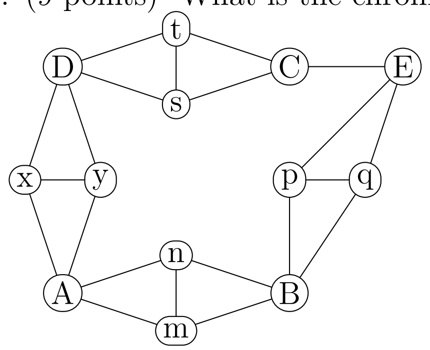
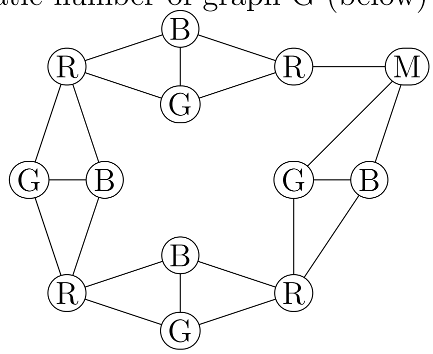
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is four. The picture above shows that it can be colored with four colors (upper bound).

Suppose we try to color it with only three colors. Suppose we color C, t, s with R, B, and G (respectively). Then D must be colored R. This forces x and y to be B and G (in either order). A is again R. And so on around the circle until we get to E. E has neighbors of all three colors. So four colors are required (lower bound).

2. (6 points) Check the (single) box that best characterizes each item.

The 10 students wouldn't fit into John's van. 10 is _____ how many students the van can carry.

an upper bound on [√] exactly [ ]
a lower bound on [ ] not a bound on [ ]

$\sum_{i=1}^{p-1} \frac{i}{p}$

$\frac{p(p-1)}{2}$ [ ] $\frac{p(p+1)}{2}$ [ ] $\frac{(p+1)}{2}$ [ ] $\frac{(p-1)}{2}$ [√]

Chromatic number of a graph with no cycles and at least one edge

1 [ ] 2 [√] 3 [ ] can't tell [ ]

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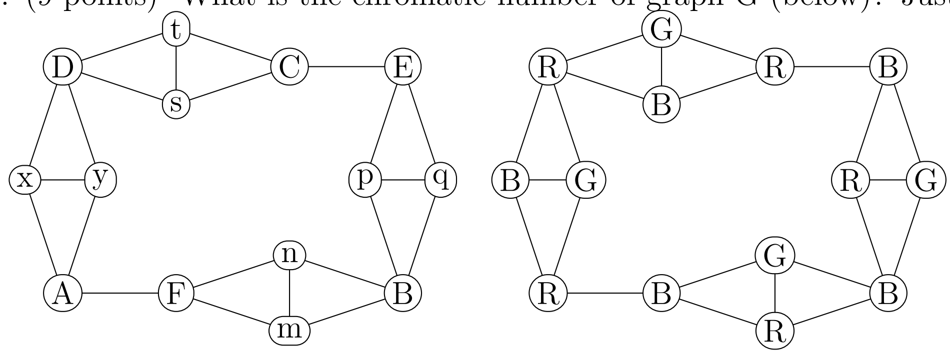

Solution: The chromatic number is three. The picture above shows that it can be colored with three colors (upper bound). Since it contains triangles, we also have a lower bound of three.

2. (6 points) Check the (single) box that best characterizes each item.

W_7 is a subgraph of graph H. 7 is ______ the chromatic number of H.

an upper bound on	☐	exactly	☐
a lower bound on	☐	not a bound on	$\surd$

$$\sum_{k=0}^{n-1} \frac{1}{2^k}$$

$1-(\frac{1}{2})^{n-1}$	☐	$2-(\frac{1}{2})^{n}$	☐	$1-(\frac{1}{2})^{n}$	☐	$2-(\frac{1}{2})^{n-1}$	$\surd$

Suppose I want to estimate $\frac{103}{20}$.
3 is ______

an upper bound	☐	an exact answer	☐
a lower bound	$\surd$	not a bound on	☐

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1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} \ : \ |x+1| \leq 2\}$$
$$B = \{w \in \mathbb{R} \ : \ w^2 + 2w - 3 \leq 0\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution: $A \subseteq B$: Let x be a real number and suppose $x \in A$. Then $|x+1| \leq 2$. Therefore, $-2 \leq x+1 \leq 2$ so $-3 \leq x \leq 1$. Therefore $x+3 \geq 0$ and $x-1 \leq 0$. So $x^2+2x-3 = (x+3)(x-1) \leq 0$. So $x \in B$.

$B \subseteq A$: Let x be a real number and suppose $x \in B$. Then $x^2 + 2x - 3 \leq 0$. Factoring this polynomial, we get $(x+3)(x-1) \leq 0$. So $(x+3)$ and $(x-1)$ must have opposite signs. Since $x+3 > x-1$, it must be the case that $x+3 \geq 0$ and $x-1 \leq 0$. Therefore, $-3 \leq x+1 \leq 1$. So $|x+1| \leq 2$, and therefore $x \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with no edges

1 [√] 2 [ ] 3 [ ] can't tell [ ]

Brandon fit 14 buns into the steamer basket. 14 is ____ how many on how many buns the basket can hold.

an upper bound on [ ] exactly [ ]
a lower bound on [√] not a bound on [ ]