CS 173, Fall 2016
Examlet 8, Part A

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FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(20 points) Let function $f : \mathbb{N} \to \mathbb{Z}$ be defined by

$f(0) = 2$

$f(1) = 7$

$f(n) = f(n-1) + 2f(n-2)$, for $n \geq 2$

Use (strong) induction to prove that $f(n) = 3 \cdot 2^n + (-1)^{n+1}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that, for any integer $n \geq 8$, there are non-negative integers p and q such that $n = 3p + 5q$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that the hypercube Q_2 is a 4-cycle, and that Q_n consists of two copies of Q_{n-1} plus edges connecting corresponding nodes. A *Hamiltonian cycle* is a cycle that visits each node exactly once, except obviously for when it returns to the starting node at the end. Use (strong) induction to show Q_n has Hamiltonian cycle for any natural number $n \geq 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$f(1) = 0$ $f(2) = 12$

$f(n) = 4f(n-1) - 3f(n-2)$, for $n \geq 3$

Use (strong) induction to prove that $f(n) = 2 \cdot 3^n - 6$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) A Zellig graph has $2n$ nodes arranged in a circle. Half of the nodes have label 1 and the other half have label -1. As you move clockwise around the circle, you keep a running total of node labels. E.g. if you start at a 1 node and then pass through two -1 nodes, your running total is -1. Use (strong) induction to prove that there is a choice of starting node for which the running total stays ≥ 0.

Hint: remove an adjacent pair of nodes.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is an integer for all natural numbers n

Hint: $(a^n + b^n)(a+b) = (a^{n+1} + b^{n+1}) + ab(a^{n-1} + b^{n-1})$, for any real numbers a and b.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: