CS 173, Fall 2016
Examlet 8, Part B

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LAST:

Discussion: **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

(10 points) Suppose we have a function F defined (for n a power of 2) by

$$F(2) = c$$
$$F(n) = F(n/2) + n \text{ for } n \geq 4$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n\frac{1}{2^i}$$

Finish finding the closed form for F. Show your work and simplify your answer.

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$
\begin{aligned}
g(1) &= c \\
g(n) &= 4g(n/2) + d \text{ for } n \geq 2
\end{aligned}
$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The number of edges in the
4-dimensional hypercube Q_4 5 ☐   12 ☐   32 ☐   64 ☐

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1. (8 points) Suppose we have a function f defined by

$$
\begin{aligned}
f(1) &= 5 \\
f(n) &= 3f(n/2) + n^2 \text{ for } n \geq 2
\end{aligned}
$$

Express $f(n)$ in terms of $f(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) The n-dimensional hypercube Q_n has an Euler circuit.

always ☐   sometimes ☐   never ☐

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(10 points) Suppose we have a function F defined (for n a power of 3) by

$$\begin{aligned} F(1) &= 5 \\ F(n) &= 3F(n/3) + 7 \text{ for } n \geq 3 \end{aligned}$$

Your partner has already figured out that

$$F(n) = 3^k F(n/3^k) + 7 \sum_{p=0}^{k-1} 3^p$$

Finish finding the closed form for F. Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^{n} r^k = \dfrac{r^{n+1} - 1}{r - 1}$

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(10 points) Suppose we have a function g defined (for n a power of 4) by

$$g(1) = c$$
$$g(n) = 2g(n/4) + n \text{ for } n \geq 4$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $f(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

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1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(9) &= 5 \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 27 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 3^k g(n/3^k) + kn$$

Finish finding the closed form for g. Show your work and simplify your answer.

2. (2 points) Check the (single) box that best characterizes each item.

$f(n) = n!$ can be defined recursively
by $f(0) = 1$, and
$f(n+1) = (n+1)f(n)$ for $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐
all integers ...