CS 173, Fall 2016
Examlet 9, Part A

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(18 points) If T is a binary tree with root R, then Heft(T) is defined to be

- 0 if R is a leaf
- m if R has one child subtree T', with Heft(T') equal to m
- 1+m if R has two child subtrees V and W, with Heft(V) and Heft(W) both equal to m
- otherwise, the maximum Heft of R's two child subtrees.

Use (strong) induction to prove that a binary tree T with Heft(T)=p has at least 2^p leaves

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Recall that a node in a full binary tree is either a leaf or has exactly two children. A Happy tree is a full binary tree such the two child subtrees of each internal node have heights that differ by at most one. Prove that every Happy tree of height h contains at least F_{h+1} nodes, where F_k is the kth Fibonacci number. (Recall: $F_0 = 0$, $F_1 = F_2 = 1$)

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) A Strange tree is a 3-ary tree (i.e. nodes have between 0 and 3 children) whose nodes are labelled with natural numbers such that

- Leaf nodes have label 1.
- The label on an internal node is the sum of the labels on its children, plus one.

Use (strong) induction to prove that the root node of any Strange tree has label $\leq \frac{1}{2}(3^{h+1} - 1)$ where h is the height of the tree.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Let's define a Spooky Tree to be a binary tree containing 2D points such that:

- Each leaf node contains $(1, 2)$, $(5, 7)$, or $(-1, 10)$.
- An internal node with one child labelled (a, b) has label $(a, b + 1)$.
- An internal node with two childen labelled (x, y) and (a, b) has label $(\frac{x+a}{2}, \frac{y+b}{2})$.

Use (strong) induction to prove that the point in the root node of any Spooky tree is above the line $x = y$

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Recall that a node in a full binary tree is either a leaf or has exactly two children. A Peaceful tree is a full binary tree such the two child subtrees of each internal node have heights that differ by at most one. Prove that every Peaceful tree of height h contains at least F_{h+1} nodes, where F_k is the kth Fibonacci number. (Recall: $F_0 = 0$, $F_1 = F_2 = 1$)

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Here is a grammar G, with start symbol S and terminal symbols a and p.

$$S \;\rightarrow\; S\ S \;\mid\; p\ S\ p \;\mid\; p\ p \mid\; a\ a$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar G has an even number of nodes with label p. Use $P(T)$ as shorthand for the number of p's in a tree T.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: