CS 173, Fall 2016
Examlet 9, Part B

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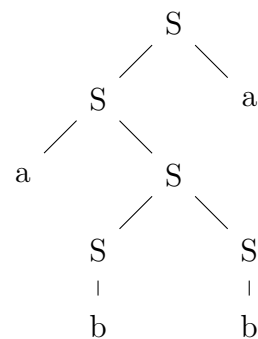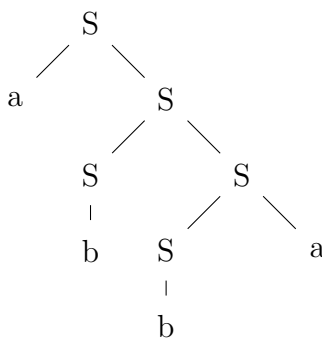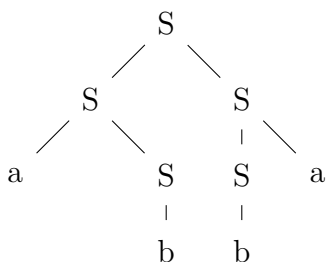
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Discussion: **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string $a\ b\ b\ a$ that match this grammar.

$$S \rightarrow S\ S \mid a\ S \mid S\ a \mid b$$

Solution:

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^{n-1} 2^k$$

$2^n + 1$ ☐ $2^n - 1$ ☐ $2^n - 2$ ☑ 2^n ☐

A tree with n nodes has

n edges ☐ $n - 1$ edges ☑ $\leq n$ edges ☐

$n/2$ edges ☐ $\log n$ edges ☐

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1. (8 points) Consider the following grammar G

$$S \rightarrow a\,S \mid a\,N$$
$$N \rightarrow N\,N \mid b\,c \mid c\,c$$

S is the only start symbol. The terminal symbol are a, b, and c.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

$a\ c\ c\ b\ c$
Solution:
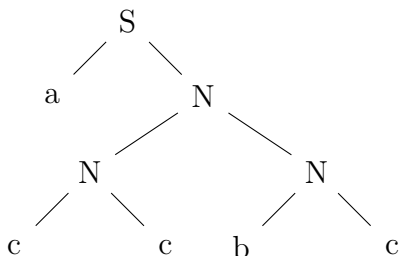
$a\ b\ c\ c\ c\ a$
Solution:
Impossible because this grammar can generate a's only at the start of the string, before all the b's and c's.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n+1} 2^k$$

$2^{n+1} + 1$ ☐ $2^{n+2} - 1$ ☑ $2^{n+2} - 2$ ☐ $2^{n+1} - 1$ ☐

The root node of a tree is an
internal node   always ☐   sometimes ☑   never ☐
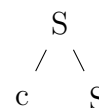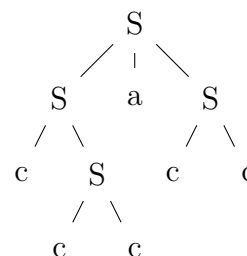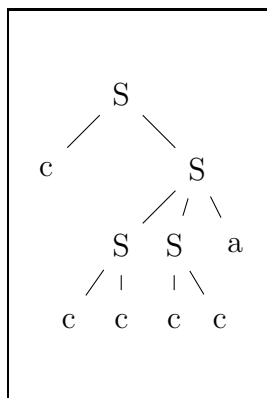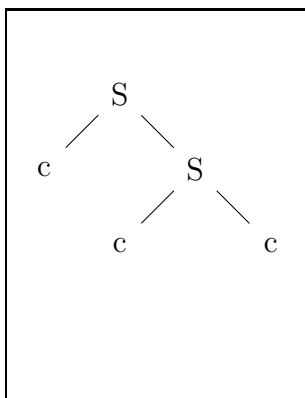
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1. (8 points)   Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \;\rightarrow\; S\,S\,a \;\mid\; c\,S \;\mid\; c\,c$$

2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string

$\emptyset$ ☐   e ☐ ϵ ☑   NULL ☐

A binary tree of height h has at least $2^h - 1$ vertices (nodes).

true ☐   false ☑

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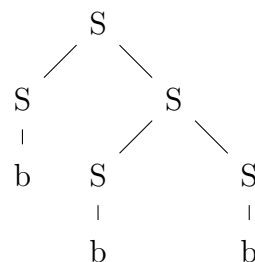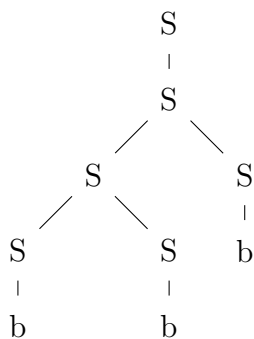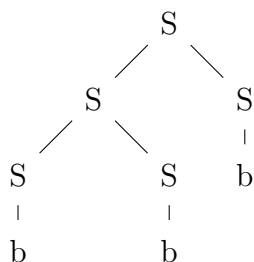NETID:

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Discussion: **Thursday**   **2**   **3**   **4**   **5**   **Friday 9**   **10**   **11**   **12**   **1**   **2**

1. (8 points) Here is a grammar with start symbol S and terminal symbol b. Draw three parse trees for the string $b\ b\ b$ that match this grammar.

$$S \rightarrow S\ S \mid S \mid b$$

Solution:

2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a full, complete tree of height h. $\leq h$ ☐ h ☐ $h+1$ ☐

$2h$ ☐ $\leq 2h$ ☑

$\displaystyle\sum_{k=1}^{n+1} 2^k$ $2^{n+1}+1$ ☐ $2^{n+2}-1$ ☐ $2^{n+2}-2$ ☑ 2^n-2 ☐   '

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Discussion: **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

1. (8 points) Consider the following grammar G

    $S \rightarrow S \, b \, S \mid a \mid c \, d$

    S is the only start symbol. The terminal symbols are a, b, c, and d.
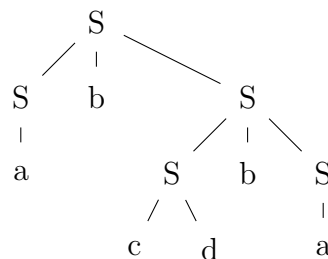
    Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

    $a \, a \, a \, c \, d$
    Solution: In grammar G, making strings with more than two leaves requires using the first rule (SbS) which produces a b. This string can't be generated by G because it is more than two characters long with no b in it.

    $a \, b \, c \, d \, b \, a$
    Solution:

2. (4 points) Check the (single) box that best characterizes each item.

    The level of a leaf node in a tree of height h.

    0 ☐ 1 ☐ $h - 1$ ☐ $\leq h$ ☑ h ☐

    $\displaystyle\sum_{k=0}^{n-1} 2^k$ $2^n - 2$ ☐ $2^n - 1$ ☑ $2^{n-1} - 1$ ☐ $2^{n+1} - 1$ ☐

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1. (8 points)   Consider the following grammar G

   $$S \to b\,S\,a \mid b\,S\,b \mid c$$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

b b c a b a b	*b a b c b b b*
Solution: This is impossible. All strings produced by G have the (single) c in the exact middle of the string.	**Solution:** This is impossible. In strings produced by G, all a's occur after the c.

2. (4 points) Check the (single) box that best characterizes each item.

   The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as different.

n	☐	$2n$	☐	$\frac{n(n-1)}{2}$	☐
$n(n-1)$	✓	n^2	☐		

   The chromatic number of a full 3-ary tree

1	☐	2	☐	≤ 2	✓
3	☐	≤ 3	☐	can't tell	☐