CS 173, Fall 2016
Examlet 10, Part A

NETID:

FIRST: **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

(15 points) Use (strong) induction to prove the following claim:

Claim: $(2n)!^2 < (4n)!$ for all positive integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1 + nx$.

Let x be a real number with $x > -1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$

You may use the fact that $\sqrt{n+1} \geq \sqrt{n}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$.

Use (strong) induction to prove the following claim:

Claim: For every integer $n \geq 2$, $\displaystyle\prod_{p=1}^{n} \frac{2p-1}{2p} > \frac{1}{2\sqrt{n}}$

You may use the fact that $\sqrt{2} > 1.4$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Recall the following fact about real numbers

Triangle Inequality: For any real numbers x and y, $|x + y| \leq |x| + |y|$.

Use this fact and (strong) induction to prove the following claim:

Claim: For any real numbers $x_1, x_2, \ldots, x_n$ ($n \geq 2$), $|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: $\displaystyle\sum_{k=n+1}^{2n} \frac{1}{k} \geq \frac{7}{12}$, for any integer $n \geq 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: recall that if $x \leq y$, then $\frac{1}{y} \leq \frac{1}{x}$