CS 173, Fall 2016
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(8) = 5 \qquad T(n) = 3T(n-2) + c$

   (a) The height: $\frac{n}{2} - 4$

   (b) The number of nodes at level k: 3^k

   (c) Value in each node at level k: c

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\quad 7^n$ $\quad 100 \log n$ $\quad n \log(n^7)$ $\quad 2^{3n}$ $\quad \log(2^n)$ $\quad (n^3)^7$

   Solution:

   $100 \log n \ll \log(2^n) \ll n \log(n^7) \ll (n^3)^7 \ll 7^n \ll 2^{3n} \ll 42n!$

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1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

   Solution:

   This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 2T(n/2) + n$	$\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☑ $\Theta(n^2)$ ☐
$T(1) = d$ $T(n) = 2T(n/2) + c$	$\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(2^n)$ ☐
$n^{log_3 5}$ grows	faster than n^2 ☐ slower than n^2 ☑ at the same rate as n^2 ☐
Suppose $f(n)$ is $O(g(n))$. Will $g(n)$ be $O(f(n))$?	no ☐ perhaps ☑ yes ☐

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

   Solution: There are positive reals c and k such that $0 \leq f(x) \leq cg(x)$ for every $x \geq k$.

2. (8 points) Check the (single) box that best characterizes each item.

   $T(1) = d$
   $T(n) = 3T(n/3) + c$

   $\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

   $T(1) = d$
   $T(n) = 2T(n/2) + c$

   $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(2^n)$ ☐

   2^n

   $O(n!)$ ☑ $\Theta(n!)$ ☐ neither of these ☐

   $n^{log_2 3}$ grows

   faster than n^2 ☐ slower than n^2 ☑

   at the same rate as n^2 ☐

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height: $\frac{n-1}{2}$

   (b) The number of leaves: $n(n-2)(n-4)\ldots 5 \cdot 3 \cdot 1$

   (c) Value in each node at level k: $n - 2k$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^{31} \qquad n^3 \qquad 100n\log n \qquad 3^n \qquad 3\log(n^3) \qquad 7n! + 2 \qquad 173n - 173$

   Solution:

   $3\log(n^3) \ll 173n - 173 \ll 100n\log n \ll n^3 \ll 2^n + 3^{31} \ll 3^n \ll 7n! + 2$

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(1) = 7$ $\quad\quad T(n) = 2T\left(\frac{n}{4}\right) + n$

   (a) The height: $\log_4(n)$

   (b) Number of leaves: $2^{\log_4 n} = n^{1/2} = \sqrt{n}$
   [Ok to stop simplifying at $n^{1/2}$.]

   (c) Total work (sum of the nodes) at level k (please simplify):
   There are 2^k nodes at level k. Each of these nodes contains the value $n/4^k$. So the total work is $2^k \cdot n/4^k = n/2^k$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \quad n\log(17n) \quad \sqrt{n}+18 \quad 8n^2 \quad 2^n+n! \quad 2^{\log_4 n}+5^n \quad 0.001n^3+3^n$

   Solution:

   $\sqrt{n} + 18 \ll n \ll n\log(17n) \ll 8n^2 \ll 0.001n^3 + 3^n \ll 2^{\log_4 n} + 5^n \ll 2^n + n!$

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NETID:

FIRST: | **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

   Solution: This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

   $T(1) = d$
   $T(n) = 3T(n/3) + c$
   $\Theta(n)$ [√] $\Theta(n \log n)$ [ ] $\Theta(n^2)$ [ ] $\Theta(2^n)$ [ ]

   $T(1) = d$
   $T(n) = T(n/2) + n$
   $\Theta(\log n)$ [ ] $\Theta(n)$ [√] $\Theta(n \log n)$ [ ] $\Theta(n^2)$ [ ]

   Dividing a problem of size n into m subproblems, each of size n/k, has the best big-Θ running time when
   $k < m$ [ ] $k = m$ [ ]
   $k > m$ [√] $km = 1$ [ ]

   $n^{1.5}$ is
   $\Theta(n^{1.414})$ [ ] $O(n^{1.414})$ [ ] neither of these [√]