CS 173, Fall 2016
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(8) = 5$ $\qquad$ $T(n) = 3T(n-2) + c$

   (a) The height:

   (b) The number of nodes at level k:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\quad$ 7^n $\quad$ $100 \log n$ $\quad$ $n \log(n^7)$ $\quad$ 2^{3n} $\quad$ $\log(2^n)$ $\quad$ $(n^3)^7$

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1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 2T(n/2) + n$ $\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(2^n)$ ☐

$n^{log_3 5}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$? no ☐ perhaps ☐ yes ☐

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/3) + c$ — $\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$ — $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(2^n)$ ☐

2^n — $O(n!)$ ☐ $\Theta(n!)$ ☐ neither of these ☐

$n^{log_2 3}$ grows — faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height:

   (b) The number of leaves:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^{31}$ $\quad n^3$ $\quad 100n \log n$ $\quad 3^n$ $\quad 3\log(n^3)$ $\quad 7n! + 2$ $\quad 173n - 173$

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(1) = 7$ $\qquad$ $T(n) = 2T\left(\frac{n}{4}\right) + n$

   (a) The height:

   (b) Number of leaves:

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $\qquad$ $n\log(17n)$ $\qquad$ $\sqrt{n}+18$ $\qquad$ $8n^2$ $\qquad$ $2^n+n!$ $\qquad$ $2^{\log_4 n}+5^n$ $\qquad$ $0.001n^3+3^n$

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1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/3) + c$

$\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(2^n)$ ☐

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

Dividing a problem of size n into m subproblems, each of size n/k, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

$n^{1.5}$ is

$\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☐