CS 173, Fall 2016
Examlet 12, Part B

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Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(9 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $f(a) = f(b) \cap f(c)$. Express a in terms of b and c. Briefly justify your answer.

Solution: Every element of $f(b)$ contains all multiples of b and $f(c)$ contains multiples of c. So $f(a)$ must contain all numbers that are multiples of both b and c. a is the smallest element of $f(a)$. So $a = lcm(b, c)$.

(6 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).

- $\binom{17}{5}$ ☐
- $\binom{20}{4}$ ☐
- $\binom{20}{3}$ ☑ √
- $\binom{17}{4}$ ☐
- $\binom{21}{4}$ ☐
- $\frac{17!}{4!}$ ☐

$\binom{0}{0}$

- -1 ☐
- 0 ☐
- 1 ☑ √
- 2 ☐
- undefined ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$

- always ☐
- sometimes ☑ √
- never ☐

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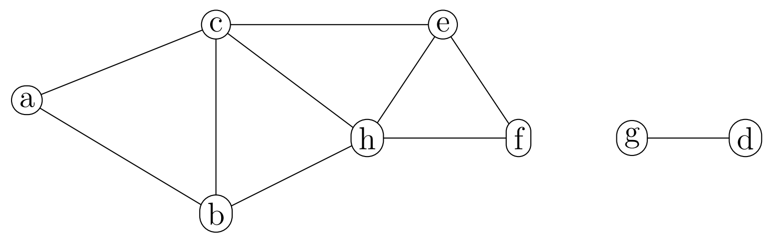
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|E| =$ **Solution:** 10

$f(d) =$ **Solution:** $\{gd\}$

$f(h) =$ **Solution:** $\{bh, ch, eh, fh\}$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: T is not a partition of E. T does not contain the empty set (good) and each edge in E is in some member of T (good). However, each edge in E is in two members of T, so there is partial overlap among the members of T (bad).

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

Solution: $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

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Let $f : \mathbb{Z}_{12} \to \mathbb{P}(\mathbb{Z}_{12})$ be defined by $f(x) = \{y \in \mathbb{Z}_{12} \mid y^2 = x\}$.

Let $S = \{f(x) \mid x \in \mathbb{Z}_{12}\}$.

(6 points) Fill in the following values. (You can write elements of $\mathbb{Z}_{12}$ as plain integers, without brackets.)

$f(4) =$

Solution: $\{2, 4, 8, 10\}$

$f(7) =$

Solution: $\emptyset$

$S =$

Solution: $\{\{2, 4, 8, 10\}, \{0, 6\}, \{1, 5, 7, 11\}, \{3, 9\}, \emptyset\}$

(7 points) Is S a partition of $\mathbb{Z}_{12}$? For each of the conditions required to be a partition, briefly explain why S does or doesn't satisfy that condition.

Solution: No. S covers all of $\mathbb{Z}_{12}$ and has no partial overlap. However, it can't be a partition because it contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set,
$\{A\}$ is a partition of A. always [√] sometimes [ ] never [ ]

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(9 points) Suppose that A and B are sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

Solution: No. Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$. Then $C_A = \{\{1, 2\}, \{3\}\}$ is a partition of A and $C_A = \{\{1\}, \{2, 4\}\}$ is a partition of B. But $C_A \cup C_B = \{\{1, 2\}, \{3\}, \{1\}, \{2, 4\}\}$ has partial overlap, so it can't be a partition of $A \cup B = \{1, 2, 3, 4\}$.

(6 points) Check the (single) box that best characterizes each item.

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true [√] false [ ]

Pascal's identity states that $\binom{n}{k}$ is equal to $\binom{n-1}{k} + \binom{n-1}{k-1}$ [√] $\binom{n-1}{k} + \binom{n-1}{k+1}$ [ ] $\binom{n-1}{k} + \binom{n-2}{k}$ [ ]

$\{\{a, b\}, c\} = \{a, b, c\}$ true [ ] false [√]

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Suppose that $A = \{2, 3, 5, 13, 17\}$. Let's define a function $F : A \rightarrow \mathbb{P}(A)$ and a set S as follows:

$$\begin{aligned} F(x) &= \{y \in A \mid y \text{ is a factor of } x\} \\ S &= \{F(x) \mid x \in A\} \end{aligned}$$

(6 points) Fill in the following values:

$F(13) =$ **Solution:** 13

$S =$ **Solution:** $\{\{2\}, \{3\}, \{5\}, \{13\}, \{17\}\}$.

(7 points) Is S a partition of A? For each of the conditions required to be a partition, briefly explain why S does or doesn't satisfy that condition.

Solution: Yes. S is a partition of A. Notice that $f(n) = \{n\}$ for all n in this particular set A. So element of A is in exactly one member of S and S cannot contain the empty set.

(2 points) State the binomial theorem.

Solution:

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

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(9 points) Suppose that $f : A \to B$ is a function. Let's define $T : B \to \mathbb{P}(A)$ by $T(m) = \{x \in A \mid f(x) = m\}$. Then let $P = \{T(m) \mid m \in B\}$. Under what conditions is P a partition of A? Briefly justify your answer.

Solution: $T(m)$ is the set of pre-images of m. Every element $x \in A$ has exactly one image in B. So it belongs to exactly one set $T(m)$. That covers two of the partition properties.

However, P will contain the empty set if f is not onto. So P is a partition if and only if f is onto.

(6 points) Check the (single) box that best characterizes each item.

Pascal's identity states that $\binom{n+1}{k}$ is equal to

$\binom{n}{k} + \binom{n}{k+1}$ ☐	$\binom{n}{k} + \binom{n-1}{k}$ ☐	$\binom{n}{k} + \binom{n}{k-1}$ ☑ √

Set B is a partition of a finite set A. Then

$\|B\| \leq 2^{\|A\|}$ ☐	$\|B\| \leq \|A\|$ ☑ √
$\|B\| = 2^{\|A\|}$ ☐	$\|B\| \leq \|A+1\|$ ☐

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).

$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☑ √
$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐