CS 173, Fall 2016
Examlet 12, Part B

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(9 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $f(a) = f(b) \cap f(c)$. Express a in terms of b and c. Briefly justify your answer.

(6 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).

$\binom{17}{5}$ ☐ $\binom{20}{4}$ ☐ $\binom{20}{3}$ ☐

$\binom{17}{4}$ ☐ $\binom{21}{4}$ ☐ $\frac{17!}{4!}$ ☐

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ undefined ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

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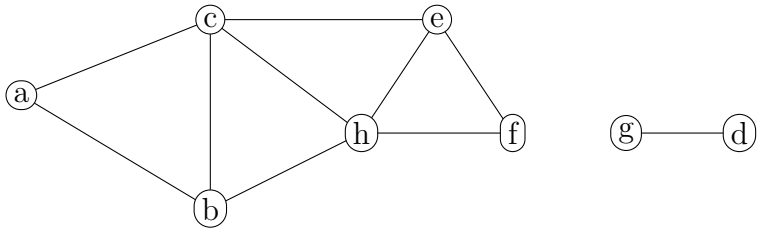
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|E| =$

$f(d) =$

$f(h) =$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

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Let $f : \mathbb{Z}_{12} \to \mathbb{P}(\mathbb{Z}_{12})$ be defined by $f(x) = \{y \in \mathbb{Z}_{12} \mid y^2 = x\}$.

Let $S = \{f(x) \mid x \in \mathbb{Z}_{12}\}$.

(6 points) Fill in the following values. (You can write elements of $\mathbb{Z}_{12}$ as plain integers, without brackets.)

$f(4) =$

$f(7) =$

$S =$

(7 points) Is S a partition of $\mathbb{Z}_{12}$? For each of the conditions required to be a partition, briefly explain why S does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set,
$\{A\}$ is a partition of A. always ☐ sometimes ☐ never ☐

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(9 points) Suppose that A and B are sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

(6 points) Check the (single) box that best characterizes each item.

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☐ false ☐

Pascal's identity states that $\binom{n}{k}$ is equal to $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☐ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

$\{\{a, b\}, c\} = \{a, b, c\}$ true ☐ false ☐

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Suppose that $A = \{2, 3, 5, 13, 17\}$. Let's define a function $F : A \to \mathbb{P}(A)$ and a set S as follows:

$$\begin{aligned} F(x) &= \{y \in A \mid y \text{ is a factor of } x\} \\ S &= \{F(x) \mid x \in A\} \end{aligned}$$

(6 points) Fill in the following values:

$F(13) =$

$S =$

(7 points) Is S a partition of A? For each of the conditions required to be a partition, briefly explain why S does or doesn't satisfy that condition.

(2 points) State the binomial theorem.

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(9 points) Suppose that $f : A \to B$ is a function. Let's define $T : B \to \mathbb{P}(A)$ by $T(m) = \{x \in A \mid f(x) = m\}$. Then let $P = \{T(m) \mid m \in B\}$. Under what conditions is P a partition of A? Briefly justify your answer.

(6 points) Check the (single) box that best characterizes each item.

Pascal's identity states that $\binom{n+1}{k}$ is equal to

☐ $\binom{n}{k} + \binom{n}{k+1}$ ☐ $\binom{n}{k} + \binom{n-1}{k}$ ☐ $\binom{n}{k} + \binom{n}{k-1}$

Set B is a partition of a finite set A. Then

☐ $|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$

☐ $|B| = 2^{|A|}$ ☐ $|B| \leq |A+1|$

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).

☐ $\binom{17}{5}$ ☐ $\binom{20}{4}$ ☐ $\binom{20}{3}$

☐ $\binom{17}{4}$ ☐ $\binom{21}{4}$ ☐ $\frac{17!}{4!}$