CS 173, Fall 16
Examlet 13, Part A

NETID:

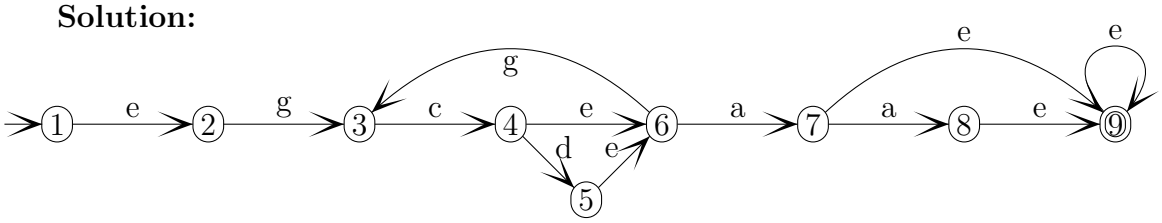
FIRST:	LAST:

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(15 points) Anne wants to model the "tunes" that her small brother is playing on his xylophone. He uses 5 notes: c, d, e, g, and a. She has observed that he plays one e, followed by one or more copies of gce or gcde, followed by one or two copies of a, followed by one or more copies of e. When he repeats gce/gcde, he switches between gce and gcde with no obvious pattern. Model this as a state diagram with one note on each edge, using no more than 11 states and, if you can, no more than 9.

Your state machine must be deterministic. That is, if you look at any state s and any action a, there is never more than one edge labelled a leaving state s.

Solution:



CS 173, Fall 16
Examlet 13, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(5 points) Let A be the set of all functions from $\{0, 1\}$ to $\mathbb{N}$. Is A countable? Briefly justify your answer.

Solution: Yes, A is countable. You can view a function from $\{0, 1\}$ to $\mathbb{N}$ as a pair of natural numbers: the image of 0 and the image of 1. We know that $\mathbb{N}^2$ is countable.

(10 points) Check the (single) box that best characterizes each item.

The interval $[2, 3]$ of the real line.	finite ☐	countably infinite ☐	uncountable ☑
$f : A \to B$ is one-to-one if and only if $\lvert A\rvert \leq \lvert B\rvert$.	true ☑	false ☐	true for finite sets ☐
The set of netIDs currently in use at U. Illinois.	finite ☑	countably infinite ☐	uncountable ☐
Every real number has a corresponding finite formula.	true ☐	false ☑	not known ☐
The set of all finite lists of integers.	finite ☐	countably infinite ☑	uncountable ☐

CS 173, Fall 16
Review, Part A

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $\gcd(a, b) > 1$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Solution:

This relation is not transitive. Consider 2, 6, and 3. Then $\gcd(2, 6) > 1$ and $\gcd(6, 3) > 1$, but $\gcd(2, 3) = 1$.

(10 points) Check the (single) box that best characterizes each item.

Putting 10 people in the canoe caused it to sink. 10 is _____ how many people the canoe can carry.

an upper bound on [√] exactly [ ]
a lower bound on [ ] not a bound on [ ]

$\neg(p \to q) \equiv \neg p \to \neg q$

true [ ] false [√]

$\{13, 14, 15\} \times \emptyset =$

$\emptyset$ [√] $\{\emptyset\}$ [ ] $\{13, 14, 15\}$ [ ]

$7 \mid 0$

true [√] false [ ]

$f : \mathbb{N} \to \mathbb{N}$,
$f(x) = 3 - x$

one-to-one [ ] not one-to-one [ ] not a function [√]

CS 173, Fall 16
Review, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(5 points) Suppose that $|A| = 3$ and $|B| = 2$. How many onto functions are there from A to B? Briefly justify or show work.

Solution: It doesn't matter what the elements of A and B are, so let's suppose that $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Two elements of A must map to the same output value, with the third element x mapping to the other output value. There are three choices for which element x is. And then there are two choices for which output value corresponds to x. So 6 onto functions total.

(10 points) Check the (single) box that best characterizes each item.

$\sum_{i=0}^{k-1}(k \cdot i + 2)$

- $\frac{k^2(k-1)}{2} + 2k$ [√]
- $\frac{k(k+1)}{2} + 2(k-1)$ [ ]
- $\frac{k^2(k+1)}{2} + 2k$ [ ]
- $\frac{k(k-1)}{2} + 2(k-1)$ [ ]

The number of edges in the 4-dimensional hypercube Q_4

- 5 [ ]
- 12 [ ]
- 32 [√]
- 64 [ ]

3^n is

- $\Theta(5^n)$ [ ]
- $O(5^n)$ [√]
- neither of these [ ]

The level of the root node in a tree of height h.

- 0 [√]
- 1 [ ]
- $h-1$ [ ]
- h [ ]
- $h+1$ [ ]

The running time of Karatsuba's algorithm is recursively defined by $T(1) = d$ and $T(n) =$

- $2T(n/2) + cn$ [ ]
- $3T(n/2) + cn$ [√]
- $4T(n/2) + cn$ [ ]
- $4T(n/2) + c$ [ ]