# CS 173, Fall 16
# Examlet 13, Part A

**NETID:**

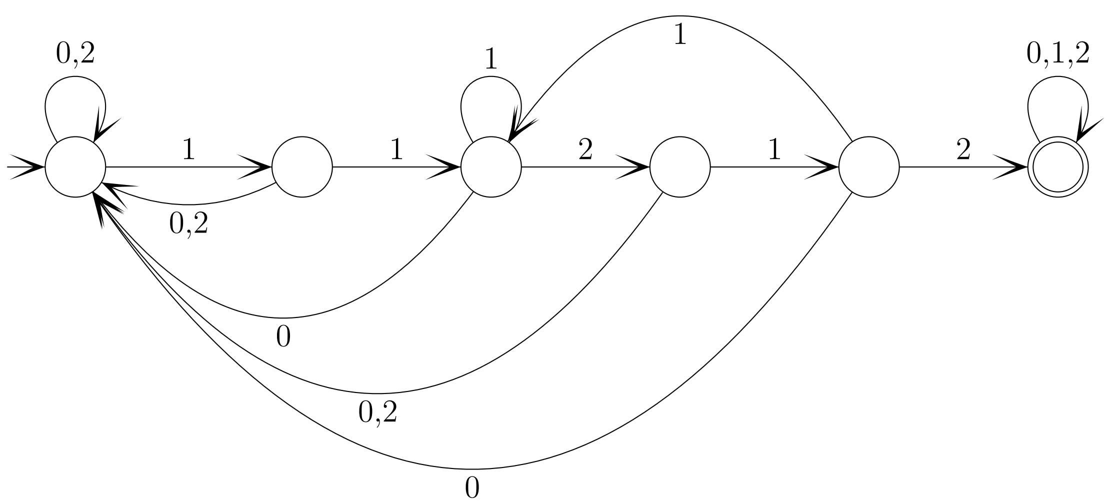
| **FIRST:** | **LAST:** |
|---|---|

**Discussion:** **Thursday** **2** **3** **4** **5** **Friday 9** **10** **11** **12** **1** **2**

(15 points) Professor Martinez needs a state machine that will recognize the sequence 11212 when typed on a keypad. Specifically, it must read any sequence of the digits 0, 1, and 2. It should move into a final state immediately after seeing 11212, and then remain in that final state as further characters come in. For efficiency, the state machine must be deterministic, i.e. if you look at any state $s$ and any action $a$, there is **exactly** one edge labelled $a$ leaving state $s$.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

**Solution:**



# CS 173, Fall 16
# Examlet 13, Part B

| NETID: |
|---|

| FIRST: | LAST: |
|---|---|

**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(5 points) Let's choose a fixed connected graph $G$ with 47 nodes. Let $A$ be the set of all walks between nodes in $G$. Is $A$ countable? Briefly justify your answer.

**Solution:** Yes, $A$ is countable. We can label the nodes and then represent each walk as a finite sequence of node labels. We know that there are only countably many finite strings of ASCII characters. Notice that $A$ is not finite, because walks can get arbitrarily long.

(10 points) Check the (single) box that best characterizes each item.

| | | | |
|---|---|---|---|
| $\|\mathbb{N}^2\| < \|\mathbb{N}^3\|$ | true ☐ | false ☑ $\surd$ | not known ☐ |
| $\|A \times A\| \geq \|A\|$ | true ☑ $\surd$ | false ☐ | true for some sets ☐ |
| The set of all polynomials with rational coefficients. | finite ☐ | countably infinite ☑ $\surd$ | uncountable ☐ |
| If $A$ is countable, then $\mathbb{P}(A)$ is countable. | always ☐ | sometimes ☑ $\surd$ | never ☐ |
| The irrational numbers | finite ☐ | countably infinite ☐ | uncountable ☑ $\surd$ |

# CS 173, Fall 16
# Review, Part A

| NETID: |
|---|

| FIRST: | LAST: |
|---|---|

**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tiger $k$, if $k$ is orange, then $k$ is large and $k$ is not friendly.

**Solution:** There is a tiger $k$ such that $k$ is not large or $k$ is friendly, but $k$ is orange.

(10 points) Check the (single) box that best characterizes each item.

$C_n$ is bipartite — always ☐ sometimes ☑ never ☐

If $xRy$ is never true, then the relation R is — symmetric ☐ antisymmetric ☐ both ☑ neither ☐

$|A \cup B| = |A| + |B|$ — true for all sets A ☐ true for some sets A ☑ false for all sets A ☐

$7 \equiv 5 \pmod 1$ — true ☑ false ☐

$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ — onto ☑ not onto ☐ not a function ☐

# CS 173, Fall 16
# Review, Part B

**NETID:**

**FIRST:** **LAST:**

**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(5 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. At each timestep, she can make one move, which adds or deletes one plant from her arsenal. If she starts with 7 wallnuts, 2 peashooters, and 10 starfruit, how many different sequences of 20 moves will get her to a configuration with 4 wallnuts, 10 peashooters, and 19 starfruit?

**Solution:** The sequence of 20 moves needs to delete 3 wallnuts, and add 8 peashooters and 10 starfruit. So we need to pick 3 moves in the sequence to be the ones that add wallnuts, and then 8 of the remaining 17 moves to be ones that add peashooters. So our number of choices is

$$\binom{20}{3}\binom{17}{8}$$

(10 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n+1} 2^k$  $2^{n+1}+1$ [ ]  $2^{n+2}-1$ [ ]  $2^{n+2}-2$ [√]  $2^n-2$ [ ]

Chromatic number of $G$  $\mathcal{C}(G)$ [ ]  $\phi(G)$ [ ]  $\chi(G)$ [√]  $\| G \|$ [ ]

$n^{log_2 3}$ grows  faster than $n^2$ [ ]  slower than $n^2$ [√]  at the same rate as $n^2$ [ ]

The diameter of a full, complete 7-ary tree of height $h$.  $\leq h$ [ ]  $h$ [ ]  $h+1$ [ ]  $2h$ [√]  $7h$ [√]  $7h+1$ [ ]

The running time of mergesort is recursively defined by $T(1) = d$ and $T(n) =$  $2T(n-1)+c$ [ ]  $2T(n-1)+cn$ [ ]  $2T(n/2)+c$ [ ]  $2T(n/2)+cn$ [√]