CS 173, Spring 2016
Examlet 2, Part A

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Discussion: Monday 9 10 11 12 1 2 3 4 5

(15 points) For any two real numbers x and y, the arithmetic mean is $M(x, y) = \frac{x+y}{2}$ and the harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Use proof by contrapositive to prove the following claim, using these definitions and your best mathematical style.

For all real numbers x and y ($x \neq -y$), if $x \neq y$, then $H(x, y) \neq M(x, y)$.

You must begin by explicitly stating the contrapositive of the claim:

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(15 points) Notice that, for any integer p, $\lfloor p \rfloor = \lfloor p + \frac{1}{2} \rfloor = p$. Using this fact and your best mathematical style, prove the following claim:

For any integer n, if n is odd, then $\left\lfloor \frac{n}{2} \right\rfloor^2 + \left\lfloor \frac{n}{2} \right\rfloor \geq \frac{1}{2} \left\lfloor \frac{n^2}{2} \right\rfloor$

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(15 points) Prove the following claim, working directly from the definitions of "remainder" and "divides", and using your best mathematical style.

For all real numbers k, m, n and r $(n \neq 0)$, if $r = \text{remainder}(m, n)$, $k \mid n$, and $k \mid r$, then $k \mid m$.

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(15 points) Prove the following claim, using your best mathematical style. Hint: look at remainders and use proof by cases. You may use the fact that if $a \mid b$, then $a \mid bc$ for any integers a, b, and c.

For any integer n, $n^4 - n^2$ is divisible by 3.

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(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a - b = nk$ for some integer n.

Claim: For all integers a, b, c, d, j and k (j and k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ and $j \mid k$, then $a + c \equiv b + d \pmod{j}$.