CS 173, Spring 2016
Examlet 2, Part B

NETID:

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Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

$7 \equiv 5 \pmod 1$	true	☑	false	☐		
$\gcd(k, 0)$	0	☐	k	☑	undefined	☐

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is true.

   There is an integer n such that $n \equiv 5 \pmod 6$ and $n \equiv 6 \pmod 7$?

   Solution: This is true. Consider $n = 41$. $41 \equiv 5 \pmod 6$ and $41 \equiv 6 \pmod 7$.

2. (6 points) Write pseudocode (iterative or recursive) for a function gcd(a,b) that implements the Euclidean algorithm. Assume both inputs are positive.

   Solution:

   ```
   gcd(a,b)
       x=a
       y=b
       while (b > 0)
           r = remainder(a,b)
           a = b
           b = r
       return r
   ```

3. (4 points) Check the (single) box that best characterizes each item.

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false ☑

   $29 \equiv 2 \pmod 9$ true ☑ false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $p \mid q$, then $s \equiv t \pmod{q}$.

   Solution: This is false.

   Informally, since q is larger than p, congruence mod q makes finer distinctions among numbers than p does.

   More formally, consider $s = 1$, $t = 4$, $p = 3$ and $q = 6$. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1609, 563)$. Show your work.

   Solution:

   $1609 - 2 \times 563 = 1609 - 1126 = 483$

   $563 - 483 = 80$

   $483 - 6 \times 80 = 3$

   $80 - 26 \times 3 = 80 - 78 = 2$

   $3 - 2 = 1$

   $2 - 2 \times 1 = 0$

   So the GCD is 1.

3. (4 points) Check the (single) box that best characterizes each item.

   $k \equiv -k \pmod{k}$ always [√] sometimes [ ] never [ ]

   $7 \mid 0$ true [√] false [ ]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all natural numbers a, b, and c if $ac \mid bc$, then $a \mid b$.

   Solution: This is false. Consider $a = 2$, $b = 3$, and $c = 0$. Then $ac \mid bc$, because $0 \mid 0$. However it's not the case that $a \mid b$, beccause $2 \nmid 3$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1012, 299)$. Show your work.

   Solution: $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So $\gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

   For any integers p and q, if $p \mid q$ then $p \leq q$. true ☐ false ☑ (√)

   $\gcd(0, 0)$ 0 ☐ k ☐ undefined ☑ (√)

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) = 1$, then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.

   Solution: This is true. If $\gcd(a, bc) = 1$, then a doesn't share any prime factors with bc. Since the prime factors of b are a subset of these, they also can't overlap with the prime factors of a. Similarly for c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   $k \equiv -k \pmod 7$ — always ☐ sometimes ☑ never ☐

   For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$. — true ☑ false ☐