CS 173, Spring 2016
Examlet 3, Part A

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Discussion: Monday 9 10 11 12 1 2 3 4 5

$A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 100\}$

$B = \{(p, q) \in \mathbb{R}^2 \mid p \geq 5\}$

$C = \{(a, b) \in \mathbb{R}^2 \mid b \leq 20\}$

Prove that $A \cap B \subseteq C$.

Hint: notice that $0 \leq (x - y)^2$. So $0 \leq (x^2 + y^2) - 2xy$.

Solution:

Let $(x, y) \in \mathbb{R}^2$. Suppose that $(x, y) \in A \cap B$. Then $(x, y) \in A$ and $(x, y) \in B$. So $x^2 + y^2 \leq 100$ and $x \geq 5$.

Since $0 \leq (x^2 + y^2) - 2xy$, we have $2xy \leq x^2 + y^2$. So then $x^2 + y^2 \leq 100$ implies that $2xy \leq 100$. So $xy \leq 50$.

Since $x \geq 5$, we can divide both sides by x to get $y \leq \frac{50}{x} \leq 10$. Therefore $y \leq 20$. So $(x, y) \in C$, which is what we needed to show.

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$A = \{\alpha(2, -4) + (1 - \alpha)(-3, 6)) \mid \alpha \in \mathbb{R}\}$

$B = \{(a, b) \in \mathbb{R}^2 \mid a \geq 1\}$

$C = \{(p, q) \in \mathbb{R}^2 \mid q \leq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let (x, y) be a 2D point and suppose that $(x, y) \in A \cap B$. Then $(x, y) \in A$ and $(x, y) \in B$.

Since $(x, y) \in A$, $(x, y) = \alpha(2, -4) + (1 - \alpha)(-3, 6))$ where α is a real number. So $x = 2\alpha - 3(1 - \alpha) = 5\alpha - 3$. And $y = -4\alpha + 6(1 - \alpha) = 6 - 10\alpha$.

Since $(x, y) \in B$, we know that $x \geq 1$. So $5\alpha - 3 \geq 1$. Therefore $\alpha \geq \frac{4}{5}$.

Substituting this into the equation for y, we get $y = 6 - 10\alpha \leq 6 - 10\frac{4}{5} = 6 - 8 = -2 \leq 0$. Since $y \leq 0$, $(x, y) \in C$, which is what we needed to show.

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$A = \{6a + 15b \ : \ a, b \in \mathbb{Z}\}$

$B = \{6p + 10q \ : \ p, q \in \mathbb{Z}\}$

$C = \{n \in \mathbb{Z} \ : \ 6 \mid n\}$

Prove that $A \cap B \subseteq C$. You may assume that if an integer is divisible by two distinct primes m and n, then it is divisible by mn.

Solution: Let n be an integer and suppose that $n \in A \cap B$. Then $n \in A$ and $n \in B$.

Since $n \in A$, $n = 6a + 15b$, where a and b are integers. So $n = 3(2a + 5b)$. $2a + 5b$ is an integer, since a and b are integers. So n is divisible by 3..

Since $n \in B$, $n = 6p + 10q$ where p and q are integers. So $n = 2(3p + 5q)$. $3p + 5q$ is an integer since p and q are integers. So n is divisible by 2.

Since n is divisible by 2 and 3, n is divisible by 6. Therefore, $n \in C$, which is what we needed to prove.

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$A = \{(x, y) \in \mathbb{R}^2 \ : \ y = x^2 - 2x - 1\}$

$B = \{(p, q) \in \mathbb{R}^2 \ : \ |p| \geq 3\}$

$C = \{(m, n) \in \mathbb{R}^2 \ : \ n \geq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y) \in \mathbb{R}^2$ and suppose that $(x, y) \in A \cap B$. Then $(x, y) \in A$ and $(x, y) \in B$.

Since $(x, y) \in A$, $y = x^2 - 2x - 1$. So $y = x(x - 2) - 1$.

Since $(x, y) \in B$, $|x| \geq 3$. There are two cases:

Case 1: $x \geq 3$. Then $x - 2 \geq 1$. So $y \geq 3 \cdot 1 - 1 = 2$.

Case 2: $x \leq -3$. Then $x - 3 \leq -5$. So $x(x - 2) \geq (-3)(-5) = 15$. Therefore $y \geq 14$.

In both cases, $y \geq 0$. So $(x, y) \in C$, which is what we needed to prove.

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$A = \{(a, b) \in \mathbb{R}^2 \ : \ |a + b| \leq 2\}$

$B = \{(x, y) \in \mathbb{R}^2 \ : \ |x - y + 7| \leq 1\}$

$C = \{(p, q) \in \mathbb{R}^2 \ : \ p \leq 0 \text{ and } q \geq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y) \in \mathbb{R}^2$ and suppose that $(x, y) \in A \cap B$. Then $(x, y) \in A$ and $(x, y) \in B$.

Since $(x, y) \in A$, $|x + y| \leq 2$. Since $(x, y) \in B$, $|x - y + 7| \leq 1$.

So $x + y \leq 2$ and $x - y + 7 \leq 1$. Adding these equations together, we get $2x + 7 \leq 3$. So $2x \leq -4$. And therefore $x \leq -2 \leq 0$.

Since $|x+y| \leq 2$, it's also the case that $-x-y \leq 2$. Adding this to $x-y+7 \leq 1$, we get $-2y+7 \leq 3$. So then $-2y \leq -4$. So $y \geq 2 \geq 0$.

So $x \leq 0$ and $y \geq 0$, so $(x, y) \in C$, which is what we needed to prove.