CS 173, Spring 2016
Examlet 3, Part B

NETID:

FIRST:

LAST:

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$.
   (π is the familiar constant.)
   true [√] false [ ] undefined [ ]

   $|A \cup B| \leq |A| + |B|$
   true for all sets A [√] true for some sets A [ ]
   false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$
   $[5]^4 = [7]^2 = [49] = [4]$
   $[5]^8 = [4]^2 = [16] = [7]$
   $[5]^{16} = [7]^2 = [49] = [4]$
   $[5]^{32} = [4]^2 = [16] = [7]$
   $[5]^{41} = [5]^{32} \cdot [5]^8 \cdot [5] = [7] \cdot [7] \cdot [5] = [49] \cdot [5] = [4] \cdot [5] = [20] = [2]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \cup (B - C) \subseteq (A \cup B) - C$

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{2, 3\}$.
   Then $A \cup (B - C) = \{1, 2\} \cup \{4\} = \{1, 2, 4\}$.
   But $(A \cup B) - C = \{1, 2, 3, 4\} - \{2, 3\} = \{1, 4\}$.
   So $A \cup (B - C) \not\subseteq (A \cup B) - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cap B$, then $x \in A$.

   true for all sets A and B [√] false for all sets A and B [ ] true for some sets A and B [ ]

   $\{1, 2\} \cap \emptyset =$

   $\emptyset$ [√] $\{(1, \emptyset), (2, \emptyset)\}$ [ ] $\{1, 2, \emptyset\}$ [ ]

   $\{\emptyset\}$ [ ] $\{1, 2\}$ [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[9]^{19} \times [8]^4$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:
   $[9]^2 = [81] = [3]$
   $[9]^4 = [3]^2 = [9]$
   $[9]^8 = [9]^2 = [3]$
   $[9]^{16} = [3]^2 = [9]$
   $[8]^2 = [64] = [-1]$
   $[8]^4 = [-1]^2 = [1]$
   So
   $[9]^{19} \times [8]^4 = [9]^{16} \cdot [9]^2 \cdot [9] \cdot [8]^4 = [9][3][9][1] = [27][9] = [1][9] = [9]$

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1. (4 points) $A = \{\text{ginger}, \text{clove}, \text{nutmeg}\}$ $B = \{\text{ginger}, \text{vanilla}, \text{pepper}\}$ $C = \{8\}$

   Solution:

   $(A \cap B) \times C = \{\text{ginger}\} \times C = \{(\text{ginger}, 8)\}$

   $(A \cap C) \times B = \emptyset \times B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A - B$, then $x \in B$.

true for all sets A and B	☐	true for some sets A and B	☑		
false for all sets A and B	☐				

   $\{1, 2\} \cup \emptyset =$

$\emptyset$	☐	$\{\emptyset\}$	☐	$\{1, 2\}$	☑
$\{(1, \emptyset), (2, \emptyset)\}$	☐	$\{1, 2, \emptyset\}$	☐	undefined	☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{22} = [8]^{16} \cdot [8]^4 \cdot [8]^2 = [3][4][9]$

   $[3][4][9] = [3][36] = [3][3] = [9]$

   So $[8]^{22} = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

   Solution: This is true. An element of $A \cap C$ must be in both A and C. I $A \subseteq B$, then it's also in B. But then it's in $B \cap C$.

2. (4 points) Check the (single) box that best characterizes each item.

$\lvert A - B\rvert = \lvert A\rvert - \lvert B\rvert$	true for all sets A		true for some sets A	√
	false for all sets A			
$\emptyset \in A$	true for all sets A		true for some sets A	√
	false for all sets A			

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{18} + [7]^4$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = [-3]^2 = [9]$

   $[7]^6 = ([7]^2)^3 = [-3]^3 = [-27] = [-1]$

   $[7]^{18} = ([7]^6)^3 = [-1]^3 = [-1] = [12]$

   So $[7]^{18} + [7]^4 = [12] + [9] = [21] = [8]$

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1. (4 points) $A = \{\text{trump}, \text{rubio}\}$ $B = \{\text{clinton}, \text{sanders}\}$

   $C = \{\ (\text{trump}, \text{clinton}),\ (\text{sanders}, \text{rubio})\ \}$

   Solution:

   $(B \times A) - C = \{(\text{clinton}, \text{trump}), (\text{clinton}, \text{rubio}), (\text{sanders}, \text{trump})\}$

   $(A \cap C) \times B = \emptyset \times B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cup B$, then $x \in A$.

   true for all sets A and B ☐ true for some sets A and B ☑

   false for all sets A and B ☐

   $\emptyset \subseteq A$

   true for all sets A ☑ true for some sets A ☐

   false for all sets A ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$