

**CS 173, Spring 2016**  
**Examlet 4, Part A**

**NETID:**

**FIRST:**

**LAST:**

**Discussion:   Monday   9   10   11   12   1   2   3   4   5**

Suppose that  $n$  is some integer  $\geq 2$ . Let's define the relation  $R_n$  on the integers such that  $aR_nb$  if and only if  $a \equiv b + 1 \pmod{n}$ . Prove the following claim

Claim: If  $R_n$  is symmetric, then  $n = 2$ .

You must work directly from the definition of congruence mod  $k$ , using the following version of the definition:  $x \equiv y \pmod{k}$  iff  $x - y = mk$  for some integer  $m$ . You may use the following fact about divisibility: for any non-zero integers  $p$  and  $q$ , if  $p \mid q$ , then  $|p| \leq |q|$ .

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Let  $A = \mathbb{Z}^+ \times \mathbb{Z}^+$ , i.e. pairs of positive integers. Consider the relation  $T$  on  $A$  defined by

$(a, b)T(p, q)$  if and only if  $aq \geq bp$

Prove that  $T$  is transitive.

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Let  $A = \mathbb{Z}^+ \times \mathbb{Z}^+$ , i.e. pairs of positive integers. Consider the relation  $T$  on  $A$  defined by

$(x, y)T(p, q)$  if and only if  $(xy)(p + q) < (pq)(x + y)$

Prove that  $T$  is transitive.

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Suppose that  $T$  is a relation on the integers which is antisymmetric. Let's define a relation  $R$  on pairs of integers such that  $(p, q)R(a, b)$  if and only if  $(a + b)T(p + q)$  and  $bTq$ . Prove that  $R$  is antisymmetric.

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Let  $A = \{(x, y) \in \mathbb{R}^2 \mid x + y = 10\}$ . Consider the relation  $T$  on  $A$  defined by

$(a, b)T(p, q)$  if and only if  $aq \geq bp$

Prove that  $T$  is antisymmetric.