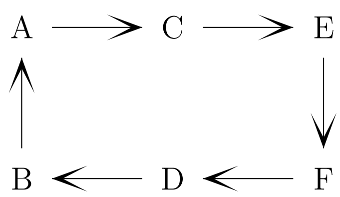
CS 173, Spring 2016
Examlet 4, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☑

Transitive: ☐

2. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R a partial order?

   Solution: Yes, R is a partial order. It's reflexive because elements are always related to themselves. It is anti-symmetric and transitive by vacuous truth.

3. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{R}$ sucn that $x \sim y$ if and only $|x - y| \in \mathbb{Z}$. List three members of [1.7].

   Solution: For example, 1.7, 2.7, and 1009.7.

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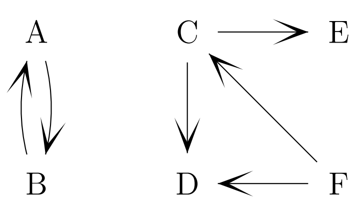

Reflexive:	☐	Irreflexive:	☑ √
Symmetric:	☐	Antisymmetric:	☐
Transitive:	☐		

2. (5 points) Can a relation with at least one related pair (i.e. at least one arrow in a diagram) be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: No, this is not possible. Suppose R is our relation and let x and y be two elements such that xRy. Then yRx because it's symmetric. Then xRx because it's transitive. But xRx means that R can't be irreflexive.

3. (5 points) Suppose that R is a relation on the integers such xRy for all integers x and y. Is R a partial order?

   Solution: No, R is not a partial order, because it's not anti-symmetric. For example, we have $2R3$ and $3R2$ but 2 and 3 aren't equal.

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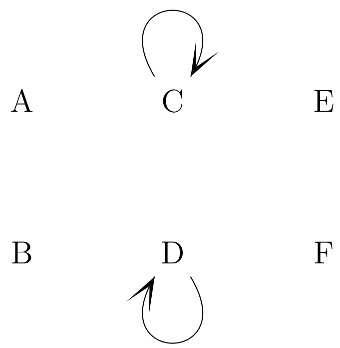

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ Antisymmetric: ☑

Transitive: ☑

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

   Solution: For any $x, y \in A$, if xRy and yRx, then $x = y$. Or for any $x, y \in A$, if xRy and $x \neq y$, then $y \not R x$.

3. (5 points) Let R be the relation on $\mathbb{Z}$ such that xRy if and only if $|x| + |y| = 2$

   Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, R is not transitive. We have $0R2$ and $2R0$, but not $0R0$.

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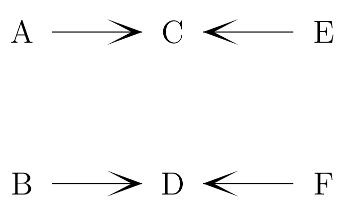
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	$\surd$
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	$\surd$		

2. (5 points) A relation is a strict partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: irreflexive, antisymmetric, transitive

3. (5 points) Let R be the relation on $\mathbb{Z}$ such that xRy if and only if $|x| + |y| = 2$

   Is R reflexive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, R is not reflexive. 2 (for example) is not related to itself.

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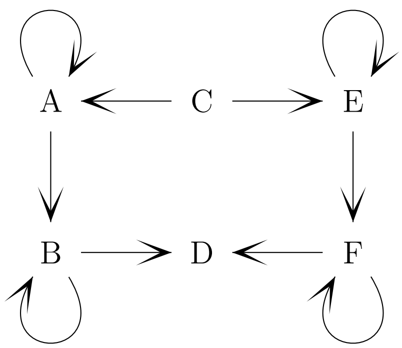
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	☑
Transitive:	☐		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be transitive.

   Solution: For any $x, y, z \in A$, if xRy and yRz, then xRz.

3. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{N}^3$ sucn that $(x, y, z) \sim (p, q, r)$ if and only $x + y + z = p + q + r$. List three members of $[(0, 0, 1)]$.

   Solution: The three coordinates need to be non-negative integers that sum to 1. So the only members of this equivalence class are $(0, 0, 1)$, $(0, 1, 0)$, and $(1, 0, 0)$.