CS 173, Spring 2016
Examlet 5, Part A

NETID:

FIRST:	**LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (10 points) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that g is one-to-one.

2. (5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.

   $B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even numbers starting with 2.

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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1. (10 points) Suppose that $f : A \to B$ and $g : B \to C$ are one-to-one. Prove that $g \circ f$ is one-to-one.

2. (5 points) What's wrong with this attempt to define $f \circ g$?

   If $f : A \to B$ and $g : B \to C$ are functions, then $f \circ g$ is the function from A to C defined by $(f \circ g)(x) = f(g(x))$.

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1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (f(x) + y, y + 3)$. Prove that g is onto.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is increasing (but perhaps not strictly increasing). Dumbledore claims that f must be one-to-one. Is he correct? Briefly explain why he is or give a concrete counter-example.

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1. (10 points) Let P be the set of pairs of positive integers. Suppose that $f : P \to \mathbb{R}^2$ is defined by $f(x, y) = (\frac{x}{y}, x + y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is one-to-one but not onto. Be specific.

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1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define the function $f : (0, \infty) \to \left(\frac{1}{3}, \infty\right)$ by $f(x) = \frac{x^2 + 2}{3x^2}$. Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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1. (10 points) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that f is onto.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ are functions. Let's define the function $f + g$ by $(f + g)(x) = f(x) + g(x)$. Adele claims that if f and g are one-to-one, then $f + g$ is one-to-one. Is this correct? Briefly explain why it is or give a counter-example.