CS 173, Spring 2016
Examlet 5, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (5 points) How many different 13-letter strings beginning with ma can be made be rearranging the characters in the word ``massachusetts''? Show your work.

   Solution: Notice that the first two characters are fixed. So there are 11 letters total to rearrange, with 4 copies of s, and two t's. So the number of possibilities is

   $$\frac{11!}{4!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

The composition of two onto functions is onto.	true	√	false			
$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 4$ (x even), $f(x) = x - 21$ (x odd)	onto		not onto	√	not a function	
$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = \lfloor x \rfloor$	one-to-one	√	not one-to-one		not a function	
$g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$	one-to-one	√	not one-to-one		not a function	
$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$	true		false	√		

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1. (5 points) 8 presidential candidates (including Bernie and Hilary) need to line up for a photo. The new editor would like Bernie and Hilary to stand next to each other. How many different ways can we arrange the eight people?

   Solution: Bernie and Hilary can be moved as a unit. So we have 7 objects to permute, so 7! permutations. However, Bernie might stand either to the left or to the right of Hilary. So the total number of possibilities is $2 \cdot 7!$.

2. (10 points) Check the (single) box that best characterizes each item.

The composition of two one-to-one functions is one-to-one.	true	√	false			
$f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$	one-to-one		not one-to-one	√	not a function	
$g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$	one-to-one	√	not one-to-one		not a function	
$g : \mathbb{R} \to \mathbb{Z}$, $g(x) = \lfloor x \rfloor$	one-to-one		not one-to-one	√	not a function	
$\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$	true	√	false			

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1. (5 points) 15 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form all of them into pairs, each pair containing one man and one woman?

   Solution: We're constructing a bijection from the women to the men (or vice versa). Since there are 15 people in each set, there are 15! bijections.

2. (10 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☑

   $f : \mathbb{Z} \to \mathbb{Z}$,
   $f(x) = x + 4$ (x even),
   $f(x) = x - 22$ (x odd)
   onto ☑ not onto ☐ not a function ☐

   $g : \mathbb{N} \to \mathbb{Z}$,
   $g(x) = x^2$
   one-to-one ☑ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z}^2 \to \mathbb{Z}^2$,
   $g(x, y) = (y, 3x)$
   one-to-one ☑ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x \neq y$ and $x + y = 0$
   true ☐ false ☑

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1. (5 points) How many different 12-letter strings can be made be rearranging the characters in the word ‘‘apalachicola’’? Show your work.

   Solution: There are 12 letters total to rearrange, with 4 copies of a, 2 copies of l, and 2 copies of c. So the total number of possibilities is

   $$\frac{12!}{4!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the ____ of f.
   domain [√] co-domain [ ] image [ ]

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$
   one-to-one [√] not one-to-one [ ] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{R}$, $g(x) = \lfloor x \rfloor$
   onto [ ] not onto [√] not a function [ ]

   $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (y, 3x)$
   one-to-one [√] not one-to-one [ ] not a function [ ]

   $\exists t \in \mathbb{Z}^+,\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = 1$
   true [√] false [ ]

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1. (5 points) To make exam grading anonymous and therefore hopefully more fair, each of the 200 students in CS 241 has been assigned a unique 3-character exam code. The character set is $\{\alpha, \beta, \gamma, \delta\}$. Use the Pigeonhole Principle to explain what's wrong with this plan.

   Solution: Since there are four distinct characters, there are $4^3 = 64$ different 3-character codes. Since there are more students than codes, the pigeonhole principle implies that there is at least one pair of students with the same code.

2. (10 points) Check the (single) box that best characterizes each item.

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x + 2.137$ — one-to-one ☐ not one-to-one ☐ not a function ☑

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, there is a color that appear on at least two vertices. — true ☑ false ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = \lfloor x \rfloor$ — onto ☑ not onto ☐ not a function ☐

   $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 4$ (x even), $f(x) = x - 21$ (x odd) — one-to-one ☐ not one-to-one ☑ not a function ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ y \leq x$ — true ☑ false ☐

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1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form as many as possible into pairs, where each pair consists of one man and one woman?

   Solution: Since we're going to run out of men first, we need to construct a one-to-one function from the men to the women. Since we have 10 input elements and 15 output elements, the number of different functions is

   $$\frac{15!}{5!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : A \to B$ is one-to-one, then — $|A| \geq |B|$ ☐ — $|A| \leq |B|$ ☑ — $|A| = |B|$ ☐

   $g : \mathbb{N} \to \mathbb{Z}$, $g(x) = |x|$ — one-to-one ☑ — not one-to-one ☐ — not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{R},\ f(p, q) = pq$ — onto ☐ — not onto ☑ — not a function ☐

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, there are two vertices with the same color. — true ☑ — false ☐

   $\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$ — true ☑ — false ☐