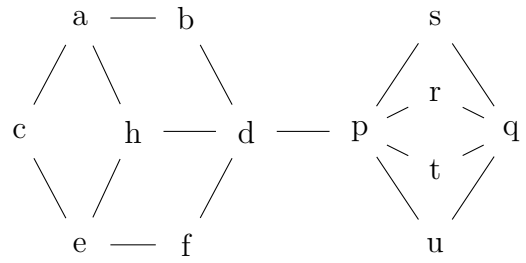
CS 173, Spring 2016
Examlet 6, Part A

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes d and p need to map to themselves. On the lefthand side, you can swap b and f (or not) and then the rest of the map is determined. On the righthand side, you can permute the nodes r, s, t, and u. So there are a total of $2 \cdot 4! = 48$ isomorphisms.

2. (5 points) The wheel graph W_{10} has 10 nodes on the rim. Is it bipartite? Briefly justify your answer.

Solution: No, it isn't bipartite. As you walk around the rim, you must assign nodes to the two subsets in an alternating manner. But there is no way to assign the hub node.

Alternatively, notice that the graph contains 3-cycles, which can't occur in bipartite graphs.

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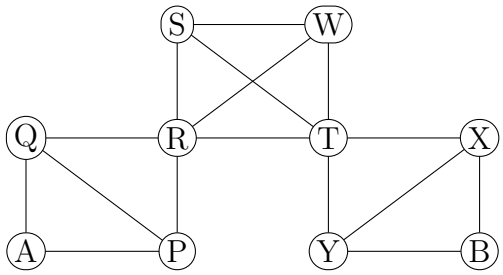

Solution: There are two choices for mapping R: to itself or to T. After that, Q and P can be interchanged (or not). And, X and Y can be interchanged (or not). Less obviously, S and W can also be interchanged. So there are $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$ isomorphisms.

2. (5 points) How many edges are in the complete bipartite graph $K_{10,5}$?

Solution: $10 \times 5 = 50$

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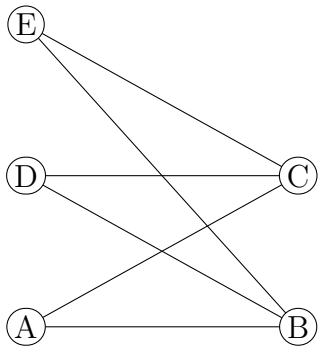
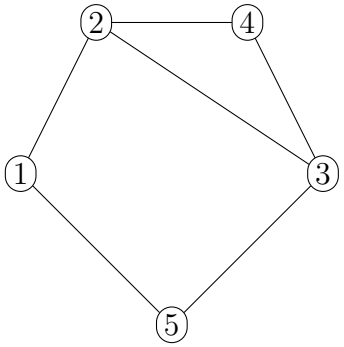
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X

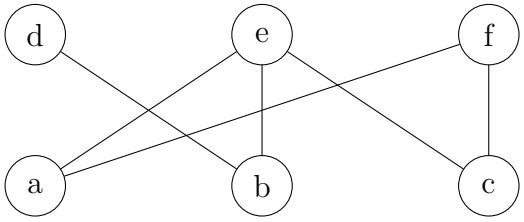


Graph Y



Solution: No, they are not isomorphic. Graph Y has a 5-cycle. Graph X is bipartite, so all of its cycles have even length.

2. (5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. All the edges go between the top set of nodes and the bottom set of nodes.

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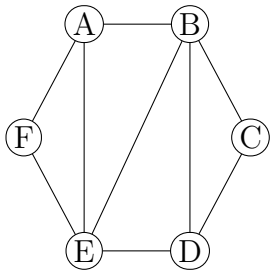
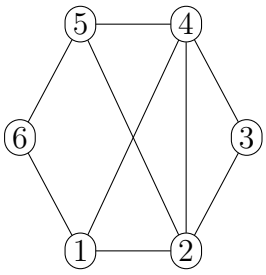
Graph X



Graph Y



Solution: No, they are not isomorphic. In graph Y, there is a degree-2 node (3) directly connected to two degree-4 nodes (2 and 4). This isn't true for either of the degree-2 nodes in graph X.

2. (5 points) The wheel graph W_{73} has 73 nodes on the rim. How many edges does it have?

Solution: It has 146 edges.

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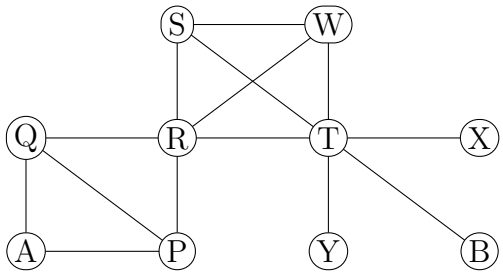

Solution: R is the only degree-5 node and T is the only degree-6 node, so they both must map to themselves.

Q and P can be interchanged (or not).

X, Y, and B can be permuted in 3! ways.

Less obviously, S and W can also be interchanged.

So there are $2 \cdot 3! \cdot 2 = 24$ isomorphisms.

2. (5 points) Does the complete graph K_8 have an Euler circuit?

Solution: No. Each node has degree 7, which is odd. You can't find an Euler circuit if there are any nodes with odd degree.

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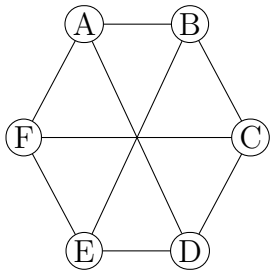
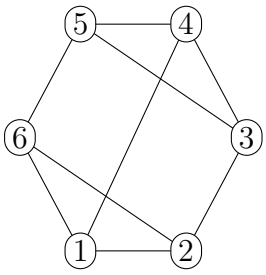
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: No, they are not isomorphic. Graph X is bipartite. (Put A, C, and E into one subset.) Graph Y contains 3-cycles, so it cannot be bipartite.

2. (5 points) Explain what a cut edge is.

Solution: An edge k in a graph is a cut edge if deleting k will separate the graph into two connected components.

Also ok is this slightly more general definition: an edge k in a graph is a cut edge if deleting k will increase the number of connected components in the graph.