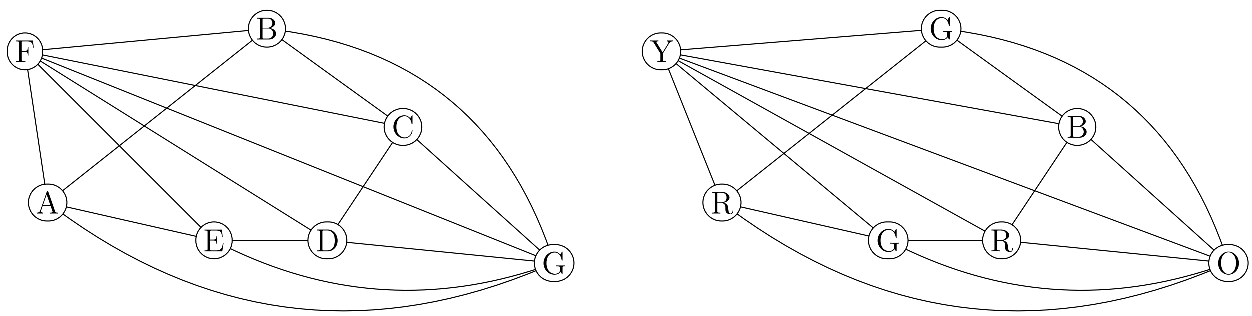
CS 173, Spring 2016
Examlet 7, Part B

NETID:

FIRST: **LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



Solution: The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors. Then the node G is connected to all six nodes in the W_5, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=3}^{n} k^7$ $\sum_{p=1}^{n-2} p^9$ ☐ $\sum_{p=1}^{n-2} (p+2)^7$ ☑ $\sum_{p=1}^{n-2} k^9$ ☐ $\sum_{p=1}^{n-2} k^7$ ☐

Suppose I want to estimate $\frac{103}{20}$. 10 is ______ an upper bound ☑ an exact answer ☐ a lower bound ☐ not a bound on ☐

The chromatic number of W_n. 2 ☐ 3 ☐ ≤ 3 ☐ ≤ 4 ☑

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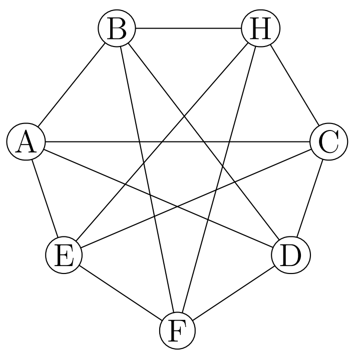
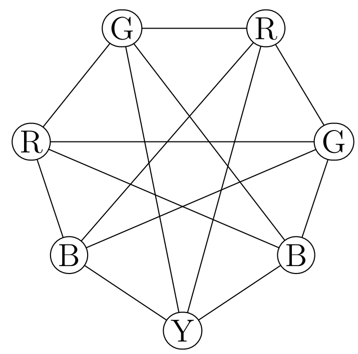
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number is 4. The picture above shows that the graph can be colored with four colors (upper bound).

To show the lower bound, let's try to color the graph with three colors. First color the triangle ABD as shown in the above picture. Then C must be colored G and E must be colored B. The colorings on C and E imply that H must be colored R.

But none of the three colors is possible for F. So three colors isn't enough, i.e. we have a lower bound of 4.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=0}^{n} \frac{1}{2^k}$	$1-(\frac{1}{2})^{n-1}$ ☐	$2-(\frac{1}{2})^{n}$ ☑	$1-(\frac{1}{2})^{n}$ ☐	$2-(\frac{1}{2})^{n-1}$ ☐
All elements of M are also elements of X.	$M = X$ ☐	$M \subseteq X$ ☑	$X \subseteq M$ ☐	
Chromatic number of a bipartite graph with at least two vertices.	1 ☐	2 ☐	3 ☐	can't tell ☑

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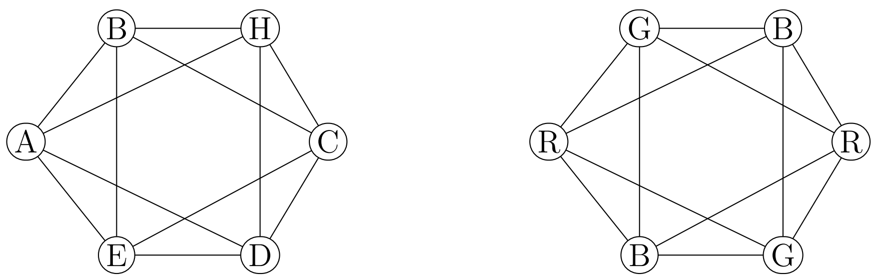
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



Solution: The chromatic number is three. The picture above shows that it can be colored with three colors (upper bound). Since it contains triangles (e.g. ABH), we also have a lower bound of three.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of G	$\mathcal{C}(G)$ ☐	$\phi(G)$ ☐	$\chi(G)$ ☑ $\surd$	$\parallel G \parallel$ ☐
$\sum_{k=1}^{n} \frac{1}{2^k}$	$1-(\frac{1}{2})^{n-1}$ ☐	$2-(\frac{1}{2})^{n}$ ☐	$1-(\frac{1}{2})^{n}$ ☑ $\surd$	$2-(\frac{1}{2})^{n-1}$ ☐
All elements of X are also elements of M.	$M = X$ ☐	$M \subseteq X$ ☐	$X \subseteq M$ ☑ $\surd$	

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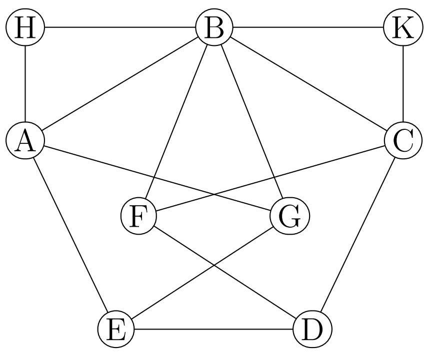
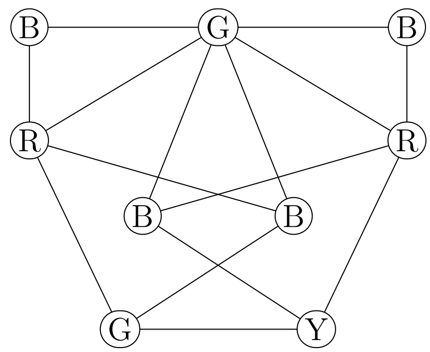
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.





Solution: The chromatic number of this graph is four.

The above picture shows that it can be colored with four colors (upper bound).

To show the lower bound, we could notice that if we delete the nodes H and K, we get a graph that we've shown to have chromatic number 4 (see section 10.3 of the textbook). If you want to argue this directly, color the triangle ABG with three colors as shown above. Then E must have the same color (G) as B. F and C must have the other two colors (in either order). This means that D has neighbors of all three colors. So three colors is not enough and therefore four is a lower bound.

2. (6 points) Check the (single) box that best characterizes each item.

10 people rowed across Lake Tahoe in my canoe. 10 is ______ how many people the canoe can carry.

an upper bound on	☐	exactly	☐
a lower bound on	☑ √	not a bound on	☐

$\sum_{i=1}^{p-1} i$

$\frac{p(p-1)}{2}$	☑ √	$\frac{(p-1)^2}{2}$	☐	$\frac{p(p+1)}{2}$	☐	$\frac{(p-1)(p+1)}{2}$	☐

The chromatic number of a graph with maximum vertex degree D

$= D$	☐	$= D + 1$	☐
$\leq D + 1$	☑ √	$\geq D + 1$	☐

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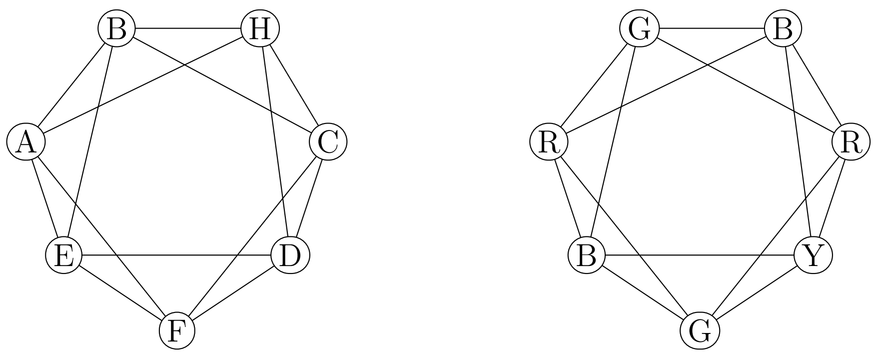
1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



Solution: The chromatic number is 4. The picture above shows that the graph can be colored with four colors (upper bound).

To show the lower bound, let's try to color the graph with three colors. First color the triangle ABH as shown in the above picture. Then C must be colored R and E must be colored B. The colorings on A and E imply that F must be colored G. But none of the three colors is possible for D. So three colors isn't enough, i.e. we have a lower bound of 4.

2. (6 points) Check the (single) box that best characterizes each item.

Leal team's bridge held 100 pounds without collapsing. 100 pounds is ____ on how much the bridge can hold.

- an upper bound on ☐
- a lower bound on ☑ √
- exactly ☐
- not a bound on ☐

$\sum_{k=3}^{n} k^7$

- $\sum_{p=1}^{n-2} p^9$ ☐
- $\sum_{p=1}^{n-2} k^7$ ☐
- $\sum_{p=1}^{n-2} k^9$ ☐
- $\sum_{p=1}^{n-2} (p+2)^7$ ☑ √

Graph H is a subgraph of W_7. 4 is a ____ the chromatic number of H.

- an upper bound on ☑ √
- a lower bound on ☐
- exactly ☐
- not a bound on ☐

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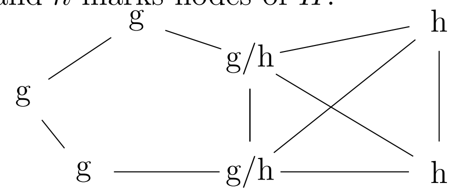
1. (11 points) If G is a graph, recall that $\chi(G)$ is its chromatic number. Suppose that G is a graph with at least one and H is another graph with at least one edge, not connected to G. Now, pick a specific edge e from G and an edge f from H and merge the two edges, creating a combined graph T. For example, suppose that G is C_5 and H is K_4. Then T might look as follows, where g marks nodes of G and h marks nodes of H.



Describe how $\chi(T)$ is related to $\chi(G)$ and $\chi(H)$, justifying your answer.

Solution: $\chi(T) = \max(\chi(G), \chi(H))$

Lower bound: G is a subgraph of T, so $\chi(T) \geq \chi(G)$. Similarly, $\chi(T) \geq \chi(H)$. So $\chi(T) \geq \max(\chi(G), \chi(H))$.

Upper bound: Suppose that $k = \max(\chi(G), \chi(H))$. We can color G with k colors, because $k \geq \chi(G)$. The merged edge in H is already colored. Because $k \geq \chi(H)$, we can extend this to a coloring of H with k colors. So $\chi(T) \leq k = \max(\chi(G), \chi(H))$.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=0}^{n-1} \frac{1}{2^k}$ $\quad 1 - (\frac{1}{2})^{n-1}$ ☐ $\quad 2 - (\frac{1}{2})^{n}$ ☐ $\quad 1 - (\frac{1}{2})^{n}$ ☐ $\quad 2 - (\frac{1}{2})^{n-1}$ ☑ √

$\tau \leq 1.3$

an upper bound on τ	√	exactly τ	☐
a lower bound on τ	☐	not a bound on τ	☐