CS 173, Spring 2016
Examlet 8, Part A

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(20 points) Suppose that $g : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$g(1) = 2$

$g(2) = 8$

$g(n) = 4(\ g(n-1) - g(n-2)\ )$

Use (strong) induction to prove that $g(n) = n2^n$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $g : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$g(1) = 1$

$g(2) = 8$

$g(n) = g(n-1) + 2g(n-2)$

Use (strong) induction to prove that $g(n) = 3 \cdot 2^{n-1} + 2(-1)^n$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is a function such that

$f(1)$ and $f(2)$ are odd

$f(n) = 2f(n-1) + 3f(n-2)$

Use (strong) induction and the definition of odd to prove that $f(n)$ is odd for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

Use (strong) induction to prove that $(\cos x + i \sin x)^n = \cos(nx) + i\sin(nx)$, for any natural number n. (i is the square root of -1.)

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $g : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$$g(1) = g(2) = 1$$

$$g(n) = 2g(n-1) + 3g(n-2)$$

Use (strong) induction to prove that $g(n) = \frac{1}{2}(3^{n-1} - (-1)^n)$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$.

Use (strong) induction to prove that $\prod_{p=2}^{n}(1 - \frac{1}{p^2}) = \frac{n+1}{2n}$ for any integer $n \geq 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: