CS 173, Spring 2016
Examlet 9, Part A

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(18 points) Suppose that a chocolate bar is a rectangle divided into a p by q grid of squares. Breaking a chocolate bar along one of the grid lines divides it into two smaller rectangular bars. Prove that it takes $pq - 1$ breaks to divide a chocolate bar into individual squares, for any choice of the order in which you choose break lines.

The induction variable is named ______ and it is the ____________________ of/in the bar.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) A Chicago tree is a binary tree whose nodes contain integers such that

- Every leaf node contains 5, 17, or 23.
- A node with one child contains the same number as its child.
- A node with two children contains the value $x(y+1)$, where x and y are the values in its children.

Use strong induction to prove that the value in the root of a Chicago tree is always positive.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) A Boston tree is a binary tree whose nodes are labelled with natural numbers such that

- If v is a leaf node, then v has label 15, 30, 20, or 40.
- If v has one child with label x, then v has label $3x$.
- If v has two children with labels x and y, then v has label $\gcd(x, y)$, i.e. the greatest common divisor of x and y. E.g. if the children have labels 30 and 40, the parent has label 10.

Use (strong) induction to prove that the root node of every Boston tree is divisible by 5. Assume we are all familiar with basic facts about divisibility, so you don't need to go all the way back to the definition.

The induction variable is named ______ and it is the __________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Suppose grammar G has start symbol S, terminal symbols a, b, c, and d, and these rules:

$$S \rightarrow S\ a\ b\ S \ \mid\ S\ d\ S \ \mid\ c\ a \ \mid\ b\ a$$

For any tree T, define $A(T)$ to be the number of nodes with label a in T, and similarly for $B(T)$ and $C(T)$. Use (strong) induction to prove that $A(T) = B(T) + C(T)$ for any tree T matching grammar G.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) A Memphis tree is a binary tree containing 2D points such that:

- Each leaf node contains $(3, 1)$, $(-2, -5)$, or $(2, 2)$.
- An internal node with one child labelled (a, b) has label $(a + 1, b - 1)$.
- An internal node with two childen labelled (x, y) and (a, b) has label $(x + a, y + b)$.

Use (strong) induction to prove that the point in the root node of any Memphis tree is on or below the line $x = y$.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Fremont trees are binary trees whose nodes are labelled with strings, such that

- Each leaf node has label `tip`, `top`, or `tack`
- If a node has one child, it has label $\alpha\alpha$ where where α is the child's label. E.g. if the child has label `top` then the parent has `toptop`.
- If a node has two children, it contains $\alpha\beta$ where α and β are the child labels. E.g. if the children have labels `tip` and `top`, then the parent has label `tiptop`.

Let S(n) be the length of the label on node n. Let L(n) be the number of leaves in the subtree rooted at n. Use (strong) induction to prove that $S(n) \geq 3L(n)$ if n is the root node of any Fremont tree.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: