CS 173, Spring 2016
Examlet 9, Part B

NETID:

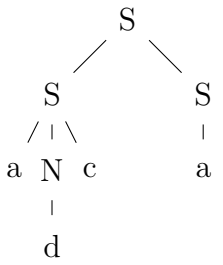
FIRST:

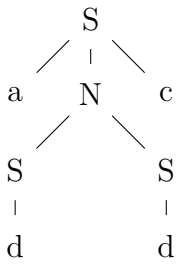
LAST:

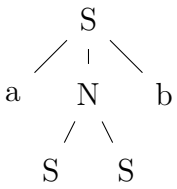
Discussion: Monday 9 10 11 12 1 2 3 4 5

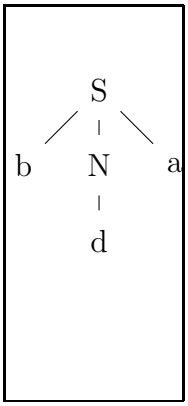
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$









2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a full, complete tree of height h.

$\leq h$		h		$h+1$	
$2h$	√	$\leq 2h$			

The level of the root node in a tree of height h.

0	√	1		$h-1$		h		$h+1$	

CS 173, Spring 2016
Examlet 9, Part B

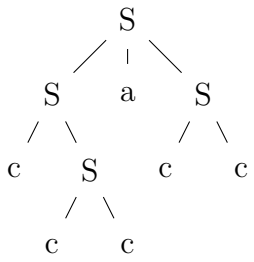
NETID:

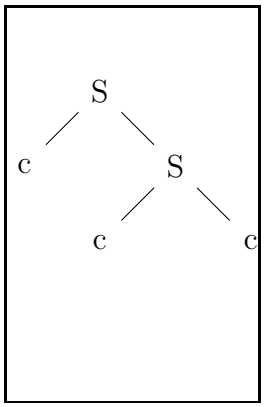
FIRST: **LAST:**

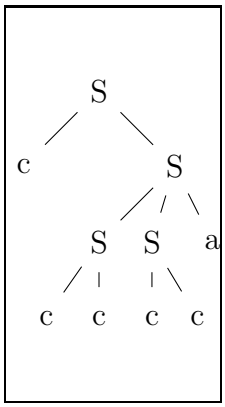
Discussion: Monday 9 10 11 12 1 2 3 4 5

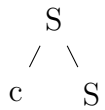
1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S\ a \mid c\ S \mid c\ c$$









2. (4 points) Check the (single) box that best characterizes each item.

The level of a leaf node in a tree of height h.

0	1	$h-1$	$\leq h$	h
			√	

$\sum_{k=0}^{n-1} 2^k$

$2^n - 2$	$2^n - 1$	$2^{n-1} - 1$	$2^{n+1} - 1$
	√		

CS 173, Spring 2016
Examlet 9, Part B

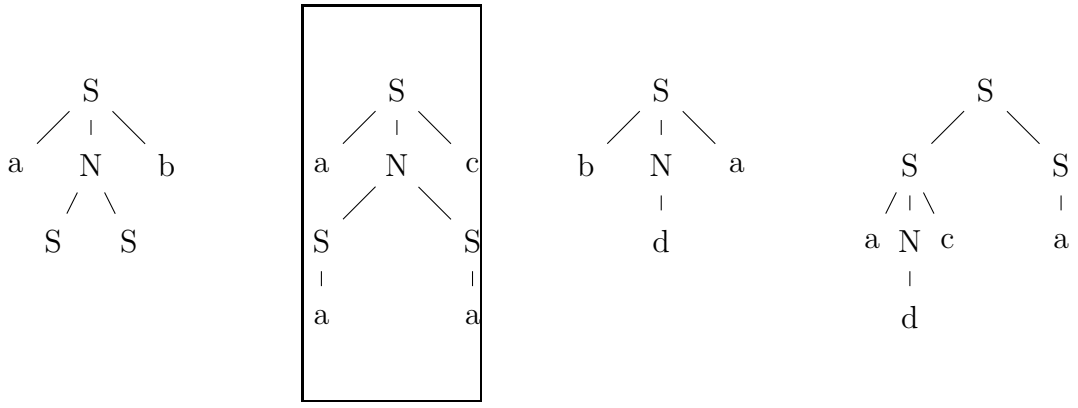
NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow a\ N\ b \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$



2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a 3-ary tree of height h

3^h	☐	$\leq 3^h$	☑
$\frac{1}{2}(3^{h+1}-1)$	☐	$3^{h+1}-1$	☐

The number of nodes in a binary tree of height h

$\geq 2^h$	☐	$2^{h+1}-1$	☐
$\leq 2^{h+1}-1$	☑	$\geq 2^{h+1}-1$	☐

CS 173, Spring 2016
Examlet 9, Part B

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (8 points) Consider the following grammar G

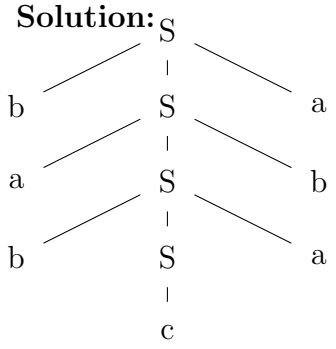
   $S \to b\ S\ a \mid a\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ b\ a\ c\ b\ a\ b$
   Solution:
   This is impossible. The grammar produces a's and b's at the same time, so its strings always have the same number of a's and b's.

   $b\ a\ b\ c\ a\ b\ a$
   Solution:



2. (4 points) Check the (single) box that best characterizes each item.

   The chromatic number of a full 3-ary tree

1	☐	2	☐	≤ 2	☑
3	☐	≤ 3	☐	can't tell	☐

   A tree with n nodes has

n edges	☐	$n-1$ edges	☑	$\leq n$ edges	☐
$n/2$ edges	☐	$\log n$ edges	☐		

CS 173, Spring 2016
Examlet 9, Part B

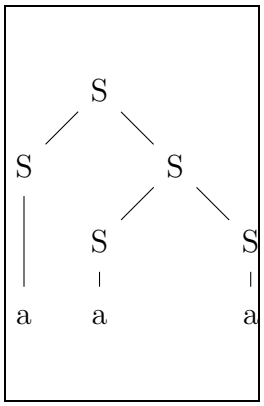
NETID:

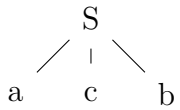
FIRST: **LAST:**

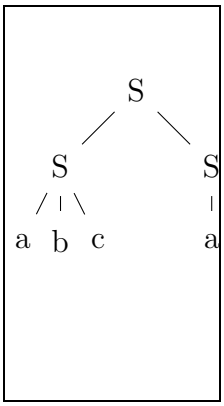
Discussion: Monday 9 10 11 12 1 2 3 4 5

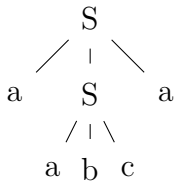
1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$









2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height h.

$\leq h$	☐	h	☐	$h+1$	☐
$2h$	☐	$\leq 2h$	☑		

Total number of leaves in a full and complete 5-ary tree of height h

5^h	☑	$\leq 5^h$	☐
$\geq 5^h$	☐	$5^{h+1}-1$	☐

CS 173, Spring 2016
Examlet 9, Part B

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (8 points) Consider the following grammar G

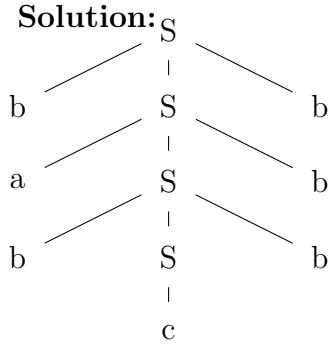
   $S \to a\ S\ b \mid b\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ a\ b\ c\ b\ b\ b$

   Solution:



   $a\ b\ c\ b\ a$

   Solution:
   This is impossible. In strings produced by G, all a's occur before the c.

2. (4 points) Check the (single) box that best characterizes each item.

   The number of paths in a tree with n nodes

n	☐	$2n$	☐	$\frac{n(n-1)}{2}$	☐
$n(n-1)$	☐	n^2	☑		

   A binary tree of height h has at least $2^h - 1$ vertices (nodes).

true	☐	false	☑